Université Toulouse 1 Capitole
Ecole d'économie de Toulouse

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Session 1

Semestre 5

Licence 3 mention Économie
mention Économie & Mathématiques
mention Économie & Droit

Epreuve : Topics in Macro 1

Date de l'épreuve : Mercredi 13 Décembre 2017

Durée de l'épreuve : 1h30

Liste des documents autorisés : aucun.

Liste des matériels autorisés : calculatrice, dictionnaire de langue.

Nombre de pages (y compris page de garde) : 5

Toulouse School of Economics, 2017-2018
L3 – Topics in Macroeconomics 1 – Loïc Batté

Final exam

<u>*Instructions:*</u>

- *There are a total of* ***22 points*** *on the test, of which* **2 bonus points**. *Please note that the number of points awarded to each question is given purely as an indication. Harder questions are indicated with a* ***(*)***.
- *You must* ***write legibly****, and* ***in a concise and precise fashion****.*
- ***Justify*** *your answers.*

1 Questions about the lectures (7 points)

1 – In this question, you are asked to discuss the assumptions made about technology in the different models seen during the lectures.

(a) **(1 point)** Explain how technology is represented in each of the following models seen in class: Solow with population growth and technological progress, Malthusian regime and Unified growth theory. In particular, is technology fixed or is there technological progress? Are the different variables representing technology exogenous or endogenous?

(b) **(1 point)** How does technology influence population in these models (if at all)?

(c) **(1 point)** How does technology influence income per capita y, or wages w, in these models (if at all)?

(d) **(1 point)** Why do we need different assumptions on technology, depending on the stage of economic growth we want to represent in the model?

2 – (1 point) Money is considered a speculative bubble: explain what this means in a few sentences.

3 – (2 points) In the model of Easter Island, which characteristics of the island explain population overshooting followed by a partial collapse, as opposed to a total collapse or a sustainable high level of population?

2 Problem – An OLG growth model (13 points + 2 bonus points)

In this problem, we shall consider an overlapping-generation (OLG) model with production, capital accumulation, technological progress and population growth. The aim of the problem is to check that an economy represented by this model attains a balanced growth path (BGP) in the long run, and verifies some Kaldor facts on the BGP.

Time is considered discrete and denoted by t. In any period, a closed economy is populated by 2 generations: the young, born in t, and the old, born in $(t-1)$. In what follows, all variables relative to an individual will be denoted with a y superscript if the variable concerns a young individual, and with an o superscript for an old individual; variables will also have a time index that represents the current period (and not the generation as in the lecture).

Total population in t writes as follows:

$$N_t = N_t^y + N_t^o \tag{1}$$

with $N_t^y = (1+n)N_t^o = (1+n)N_{t-1}^y$, and $n > 0$ a constant.

Individuals from the generation born in t seek to maximise their lifetime (or intertemporal) utility, defined as follows:

$$U(c_t^y, c_{t+1}^o) = \gamma \ln c_t^y + (1-\gamma)\ln c_{t+1}^o \tag{2}$$

with c denoting individual consumption of a final good of price 1.

It is further assumed that individuals obtain some labor income w_t by working when young,[1] and can save an amount s_t of this income to accumulate capital. In the next period, individuals (who are now old) are paid a real return r_{t+1} on this capital: the capital income which is generated is then fully consumed. Assume full depreciation of capital after one period of production,[2] so that the real interest rate r is also the return on capital net of depreciation. To avoid ambiguities, old people do not work.

The final good (which can be consumed or saved to accumulate capital) is produced according to the following aggregate production function:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \tag{3}$$

where K_t denotes aggregate capital available for production at the beginning of t, L_t is labor and A_t represents labor-augmenting technology. Assume $0 < \alpha < 1$.

[1] Individual labor supply is fixed and equal to one, so the labor income is equal to the wage rate.

[2] Full depreciation can be seen in equation (4) as well.

You are given the following laws of motion for K and A:

$$K_{t+1} = I_t \tag{4}$$
$$\Delta A_t = g A_t \tag{5}$$

where $g > 0$ is constant, and I_t denotes aggregate investment.

Advice: The problem is made of several parts, to highlight the different steps in solving the model. You can skip any part of the problem, but you may have to use results from a previous part if you do so. You are free to admit any result I give you here.

Part 1 – Individual choices of consumption and savings

1 – (1 point) Prove that the intertemporal budget constraint of this individual is:

$$c_t^y + \frac{c_{t+1}^o}{r_{t+1}} = w_t \tag{6}$$

2 – (1.5 point) Show that when the individual chooses optimally consumption and savings, one obtains the following Euler equation:

$$\frac{c_{t+1}^o}{c_t^y} = \frac{1-\gamma}{\gamma} r_{t+1} \tag{7}$$

3 – (1 point) Compute c_t^y and c_{t+1}^o, for prices w_t and r_{t+1} given.

4 – (1 point) From the previous question, show that individual savings are $s_t = (1-\gamma)w_t$. What does a high value of γ mean in terms of individual preferences? Explain the link with savings.

Part 2 – Production, factor prices and factor shares

5 – (*) (1 bonus point) Let $\tilde{k}_t = K_t/(A_t L_t)$. Assume that production is managed by a firm operating on perfectly competitive markets. Prove that $w_t = (1-\alpha)A_t \left(\tilde{k}_t\right)^\alpha$ and $r_t = \alpha \left(\tilde{k}_t\right)^{\alpha-1}$.

6 – (1 point) Factor shares are defined as the shares of each factor of production in total income. Show that the factor share of capital is α, and the factor share of labor is $(1-\alpha)$.

Part 3 – Equilibrium and dynamics of the aggregate economy

7 – (0.5 point) Write the equilibrium condition on the labor market. At which rate does the labor force grow in this model?

8 – (*) (1 point) Write the equilibrium condition on the capital market. Use it to obtain the following:

$$K_{t+1} = (1-\alpha)(1-\gamma)Y_t \tag{8}$$

Part 4 – The balanced growth path

9 – (1.5 point) Show that $\tilde{k}_t$ evolves through time according to the following law of motion:

$$\tilde{k}_{t+1} = \frac{(1-\alpha)(1-\gamma)}{(1+n)(1+g)} \left(\tilde{k}_t\right)^{\alpha} \tag{9}$$

Is there a dilution effect in this model? Explain.

10 – (0.5 point) What is the value of this variable on the balanced growth path (BGP) of the economy, denoted $\tilde{k}^*$?

11 – (*) (1 bonus point) Explain briefly why the economy always attains the BGP in the long run: $\lim_{t\to+\infty} \tilde{k}_t = \tilde{k}^*$

12 – (1 point) What is the value of g_y, the growth rate of income per capita, on the BGP? What about g_Y, the growth rate of aggregate output?

Part 5 – Checking the Kaldor facts

13 – (1 point) Does this model respect the Kaldor facts about labor productivity and the interest rate, once the economy has reached its balanced growth path? Justify your answer using previous questions.

14 – (2 points) Conclusion: how can you compare this OLG growth model to the Solow model with population growth and technological progress seen in class? Discuss both the assumptions (among which the one about savings) and the results.