Toulouse School of Economics
2016-2017

Master 1

Time series Exam - session 2
13th June 2017 - 1 hour

The exam is composed of a set of six MCQs (2 points each) and two exercises (4 points each).

MCQ

MCQ 1: Properties of stationary time series, select the right statement(s)

A A strictly stationary time series has a bounded variance

B The sum of two strong white noises is a weak white noise

C A strictly stationary time series is always a covariance stationary time series

D A strong white noise with variance σ^2 is a weak white noise

MCQ 2: Properties of stationary time series, select the right statement(s)

A A covariance stationary time series that satisfies an AR(p) representation satisfies several AR(p) representations with the different white noise error terms.

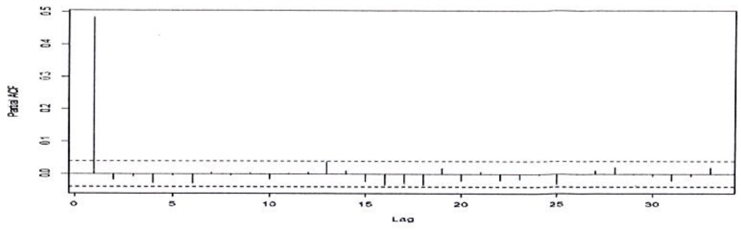
B The forecast of a MA(q) time series without deterministic term is equal to 0 for all the horizons larger or equal to q.

C An ARMA(p, q) covariance stationary time series is characterized by the fact that the partial autocorrelation function is equal to 0 for lag order strictly larger than p.

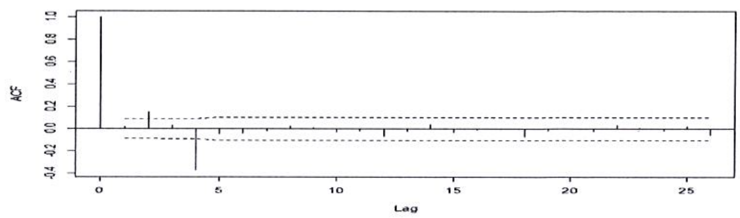
D Conditional MLE and unconditional MLE of a strictly stationary gaussian AR(p) process are asymptotically different.

MCQ 3: Which ACF or PACF is consistent with a MA(4) time series ?

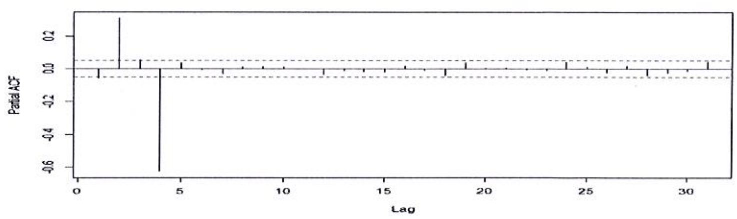
A Partial ACF



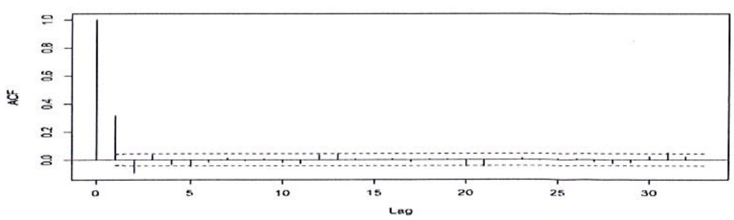
B ACF



C Partial ACF

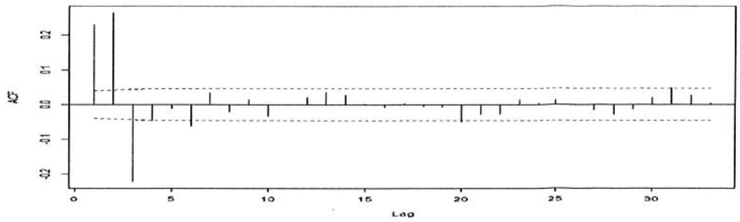


D ACF

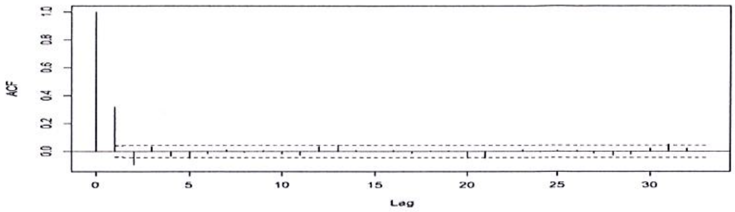


MCQ 4: Which ACF or PACF is consistent with an AR(2) time series ?

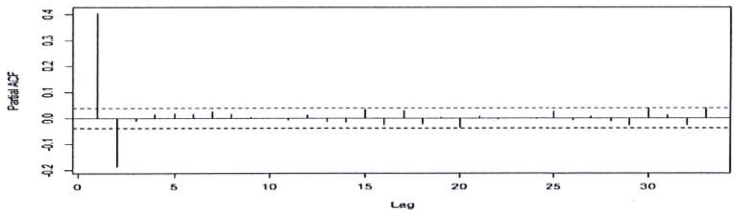
A ACF



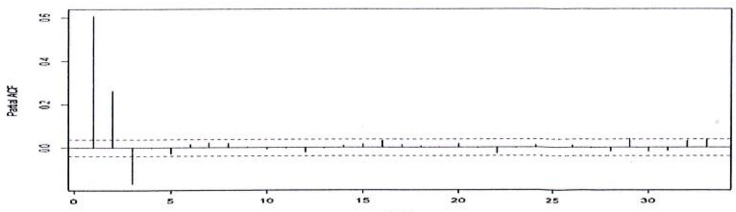
B ACF



C Partial ACF

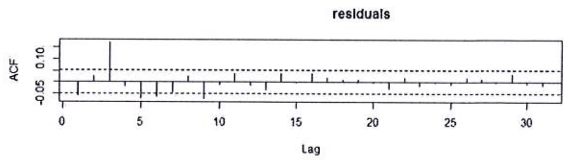


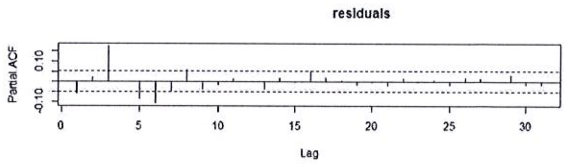
D Partial ACF



MCQ 5: From the following output (from R),

```
> xarma1<- arima(x, order=c(2,0,2))
> xarma1

Call:
arima(x = x, order = c(2, 0, 2))

Coefficients:
         ar1     ar2     ma1      ma2  intercept
      0.0787  0.5411  0.2502  -0.5312    -0.0165
s.e.  0.0585  0.0381  0.0580   0.0435     0.0518

sigma^2 estimated as 1.129:  log likelihood = -2219.58,  aic = 4449.16
> Box.test(xarma1$residuals, lag=20, type="Ljung-Box")

        Box-Ljung test

data:  xarma1$residuals
X-squared = 87.234, df = 20, p-value = 2.248e-10

> par(mfrow=c(2,1))
> acf(xarma1$residuals)
> pacf(xarma1$residuals)
```





A you decide to use this model for forecast

B you decide to increase the polynomial degrees of the MA part or AR part

C you decide to reduce the polynomial degree of the AR part

D you decide to reduce the polynomial degree of the MA part

MCQ 6: From the following output (from R),

```
> xarma<- arima(x, order=c(2,0,2))
> xarma

Call:
arima(x = x, order = c(2, 0, 2))

Coefficients:
         ar1      ar2     ma1     ma2  intercept
      0.7742  -0.0297  0.7437  0.2086     0.1942
s.e.  0.1287   0.1083  0.1266  0.0775     0.2010

sigma^2 estimated as 1.042:  log likelihood = -2160.
63,  aic = 4331.26
> Box.test(xarma$residuals, lag=20, type="Ljung-Box"
)

	Box-Ljung test

data:  xarma$residuals
X-squared = 23.324, df = 20, p-value = 0.2732
```

A you decide to use this model in a forecast exercise

B you decide to increase the polynomial degrees of the MA part or AR part

C you decide to reduce the polynomial degree of the AR part

D you decide to reduce the polynomial degree of the MA part

Exercise 1

Let $(y_t)_{t\in\mathbb{Z}}$ be a covariance stationary time series that satisfies the following equation

$$y_t = \mu + \Phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a strong white noise with variance σ^2, $\mu \in \mathbb{R}$.

1. Give the conditions under which the above equation is the canonical representation of the time series ? We consider this conditions satisfied in the sequel of the exercise.
2. Give the mean of y.
3. Give the Yule-Walker recurrence equation satisfied by the autocovariance function of y.
4. Give the forecast of y_{T+1} based on the information set $\{y_T, y_{T-1}, ...\}$.

Exercise 2

We consider two independent covariance stationary time series $(y_t)_{t\in\mathbb{Z}}$ and $(x_t)_{t\in\mathbb{Z}}$ whose canonical moving average equations are given by

$$y_t = \varepsilon_t + \theta_1\varepsilon_{t-1}$$

and

$$x_t = \eta_t + \psi_1 \eta_{t-1}$$

where $(\varepsilon_t)_{t \in \mathbb{Z}}$ and $(\eta_t)_{t \in \mathbb{Z}}$ are independent white noises with respective variances σ_ε^2 and σ_η^2. We set $z_t = y_t + x_t$.

1. Characterize the time series $(z_t)_{t \in \mathbb{Z}}$. Which kind of equation does it satisfy ?

2. Compute the forecast $z_{T+1|T}$ when $\{y_T, y_{T-1}, y_{T-2}, \ldots\}$ and $\{x_T, x_{T-1}, x_{T-2}, \ldots\}$ are known.

3. Is it equal to the forecast $z_{T+1|T}$ when $\{z_T, z_{T-1}, z_{T-2}, \ldots\}$ is known. Explain.

4. Compare the variance of the forecast error for the two above information sets.