TSE
Toulouse School of Economics
École d'économie de Toulouse

UNIVERSITÉ TOULOUSE 1 CAPITOLE

Université Toulouse 1 Capitole
Ecole d'économie de Toulouse

Année universitaire 201

2016-2017

Session 1

Semestre 2

Master 1 Economics, Econometrics & Statistics
Epreuve : Time Series
Date de l'épreuve : 30 mars 2017
Durée de l'épreuve : 1h30

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Nombre de pages (y compris page de garde) :8 pages (7+1)

Toulouse School of Economics
2016-2017

Master 1

Time series Exam
30th March 2017 - 9.00am-10.30am

The exam is composed of a set of eight MCQs (1 point each) and three exercises (4 points each).

MCQ

MCQ 1: Properties of stationary time series, select the right statement(s)

A A strictly stationary time series has a bounded variance

B A covariance stationary time series may be deterministic

C A strictly stationary time series is always a covariance stationary time series

D A weak white noise with variance σ^2 is a strong white noise

MCQ 2: Properties of stationary time series, select the right statement(s)

A The autocovariance function of a covariance stationary time series is even

B When it exists, the autocorrelation function of a strictly stationary time series takes its values in $[-1, 1]$.

C The sum of two weak noises is a weak noise.

D The spectral density of a weak white noise is a constant.

MCQ 3: Properties of stationary time series, select the right statement(s)

A A covariance stationary time series that satisfies a MA(2) representation satisfies several MA(2) representations with the same innovation process.

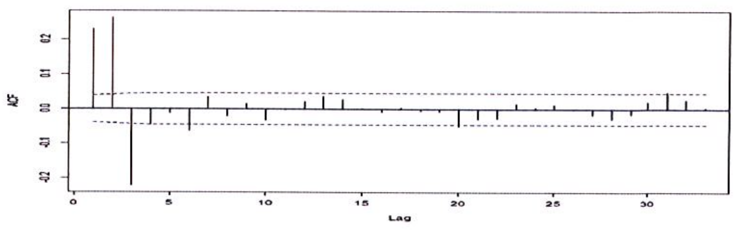
B A covariance stationary time series satisfies an AR representation. The roots of the polynomial $\Phi(L)$ associated to the AR equation must be of modulus strictly less than 1, if the weak white noise associated to this equation is the innovation process.

C The canonical representation of an ARMA(p, q) covariance stationary time series must be used when computing its forecasts.

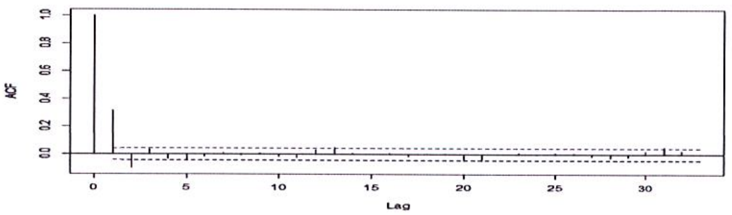
D The characterization of covariance stationary ARMA, AR and MA time series is based on more or less complicated functions of the values at various lags of the autocovariance function.

MCQ 4: Which ACF or PACF is consistent with an AR(3) time series ?

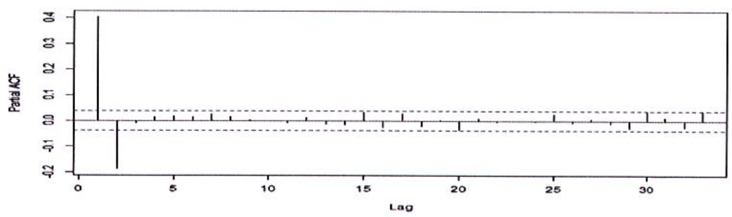
A ACF



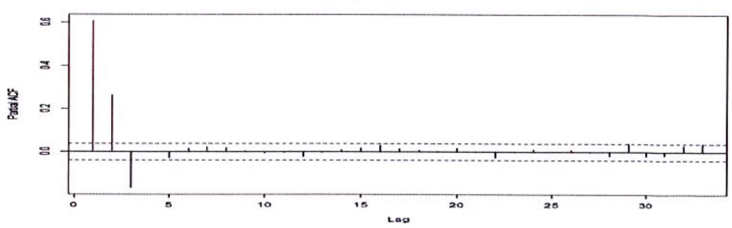
B ACF



C Partial ACF



D Partial ACF



MCQ 6: From the eacf table from R, which model do you consider for estimation ?

```
> eacf(x)
AR/MA
  0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 x x o x x o o o o o o  o  x  x
1 x x x x x o o o o o o  o  o  o
2 x x o o o o o o o o o  o  x  o
3 x x o o o o o o o o o  o  x  o
4 x x x x o o o o o o o  o  o  o
5 x x x o o o o o o o o  o  o  o
6 x x x o o o o o x o o  o  o  o
7 x x o o o o x o x o o  o  o  o
```

A ARMA(1,1)

B ARMA(2,1)

C ARMA(3,2)

D ARMA(2,2)

MCQ 7: From the following output (from R),

```
> xarma<- arima(x, order=c(3,0,2))
> xarma

Call:
arima(x = x, order = c(3, 0, 2))

Coefficients:
         ar1      ar2      ar3     ma1     ma2  intercept
      0.9854  -0.4733  -0.0018  0.0447  0.5551     0.3724
s.e.  0.0882   0.1232   0.0758  0.0763  0.0617     0.1363

sigma^2 estimated as 0.8735:  log likelihood = -676.7,  aic = 1365.4
> Box.test(xarma$residuals, lag=20, type="Ljung-Box")

	Box-Ljung test

data:  xarma$residuals
X-squared = 21.036, df = 20, p-value = 0.395
```

A you decide to use this model in a forecast exercise

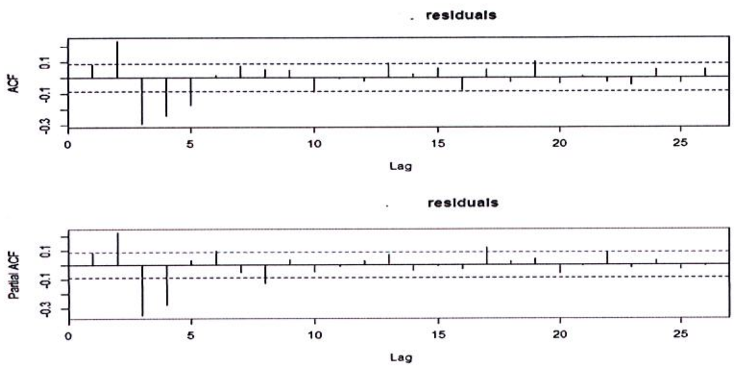
B you decide to increase the polynomial degrees of the MA part or AR part

C you decide to reduce the polynomial degree of the AR part

D you decide to reduce the polynomial degree of the MA part

MCQ 8: From the following output (from R),

```
> xarma<- arima(x, order=c(1,0,1))
> xarma

Call:
arima(x = x, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.6906  0.2834     0.3739
s.e.  0.0361  0.0374     0.1993

sigma^2 estimated as 1.166:log likelihood = -748.32,
aic = 1502.65
> Box.test(xarma$residuals, lag=20, type="Ljung-Box"
)

	Box-Ljung test

data:  xarma$residuals
X-squared = 146.01, df = 20, p-value < 2.2e-16
```



A you decide to use this model for forecast

B you decide to increase the polynomial degrees of the MA part or AR part

C you decide to reduce the polynomial degree of the AR part

D you decide to reduce the polynomial degree of the MA part

Exercise 1

Let $(y_t)_{t\in\mathbb{Z}}$ be a covariance stationary time series that satisfies the following equation

$$y_t = \mu + \Phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a strong white noise with variance σ^2, $\mu \in \mathbb{R}$.

1. Give the conditions under which the above equation is the canonical representation of the time series ? We consider these conditions satisfied in the sequel of the exercise.

2. Give the mean of y.

3. Give the Yule-Walker recurrence equation satisfied by the autocovariance function of y.

4. Give the forecast of y_{T+1} based on the information set $\{y_T, y_{T-1}, \dots\}$.

5. A statistician proposes to use an instrumental approach to estimate the parameter Φ and to use y_{t-2} as an instrument for y_{t-1}. Does that seem reasonable ? Why ?

Exercise 2

Let consider y a covariance stationary time series that satisfies the following ARMA equation

$$y_t - 2y_{t-1} = \varepsilon_t - 2\varepsilon_{t-1} + 4\varepsilon_{t-2}$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a white noise with variance σ^2.

1. Give the polynomials $\Phi(L)$ and $\Theta(L)$ that are associated to this data generating equation.

2. If this representation is not the canonical representation of the time series y, give the canonical representation of y.

3. Give the forecast function $P_{H_y^T} y_{T+h}$ as a function of the horizon h and random variables in the space spanned by the r.v.s y_s for $s \leq T$.

Exercise 3

Let consider the time series y and x that respectively satisfy the following equations

$$y_t = \Phi_1 y_{t-1} + \varepsilon_t$$

and

$$x_t = \Phi_2 x_{t-1} + \eta_t$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ and $(\eta_t)_{t\in\mathbb{Z}}$ are two strong white noises with variance σ_ε^2 and σ_η^2 and that are independent (so that $\forall(s,t) \in \mathbb{Z}, E\varepsilon_t\eta_s = 0$).

1. Give the condition(s) under which $(\varepsilon_t)_{t\in\mathbb{Z}}$ and $(\eta_t)_{t\in\mathbb{Z}}$ are the innovation processes of y and x (we assume that these conditions will be satisfied in the sequel.)

2. We define a new time series z by the following set of equations

$$\begin{aligned} z_{2t} &= y_t \\ z_{2t+1} &= x_t \end{aligned}$$

(a) Show that $\forall (s,t) \in \mathbb{Z}, E y_t x_s = 0$.

(b) Compute the variance of z. Give a condition to ensure that the variance is not time dependent. We will assume that this condition is satisfied in the sequel.

(c) Compute the autocovariance function of z. Give a condition to ensure that the covariance function is invariant by translation. We will assume that this condition is satisfied in the sequel.

(d) Give the model satisfied by z.