TSE
Toulouse School of Economics
École d'économie de Toulouse

UNIVERSITÉ TOULOUSE 1 CAPITOLE

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Session 1

Semestre 2

Master 1 Economics & Statistics

Epreuve : Mathematical Statistics 2

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Durée de l'épreuve : 2h

Liste des documents autorisés : aucun

Liste des matériels autorisés : calculatrice

Nombre de pages (y compris page de garde) : 3

Exercise 3

Let X be a *single* observation from the density defined for $x \in [0,1]$ by

$$f_\theta(x) = \theta x^{\theta-1},$$

where the parameter θ is a strictly positive real number.

a. Compute the power function of the test whose rejection region is $C = \{X \leq 1/2\}$ for testing the null hypotheses $H_0 : \theta \leq 1$ against the alternative $H_1 : \theta > 1$. Compute its level.

b. Find the uniformly most powerful test of level α $(0 < \alpha < 1)$ for testing $H_0 : \theta \leq 1$ against the alternative $H_1 : \theta > 1$.

c. Write the likelihood ratio statistic for testing the null hypotheses $H_0 : \theta = 1$ against the alternative $H_1 : \theta \neq 1$. Prove that it is a decreasing function of x. Deduce the critical region of this likelihood ratio test.

Annex

Geometric distribution. The geometric distribution Geom(p) is defined by its probability mass function, parameterized by $p \in [0,1]$:

$$f(k) = p(1-p)^k,$$

for $k \in \mathbb{N}$. The sum of r independent geometric random variables with the same parameter Geom(p) follows a negative binomial distribution $\mathcal{NB}(r,p)$ whose density is given for $k \in \mathbb{N}$ by $f(k) = C_{r+k-1}^{r-1} p^r (1-p)^k$.

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    The Negative Binomial Distribution
Description
Density, distribution function, quantile function and random generation for the negative bin
istribution with parameters size and prob.
Usage
dnbinom(x, size, prob, mu, log = FALSE)
pnbinom(q, size, prob, mu, lower.tail = TRUE, log.p = FALSE)
qnbinom(p, size, prob, mu, lower.tail = TRUE, log.p = FALSE)
rnbinom(n, size, prob, mu)
Arguments
x vector of (non-negative integer) quantiles.
q vector of quantiles.
p vector of probabilities.
n number of observations. If length(n) > 1, the length is taken to be the number required.
size target for number of successful trials, or dispersion parameter (the shape parameter of
the gamma mixing distribution). Must be strictly positive, need not be integer.
prob probability of success in each trial. 0 < prob <= 1.
mu alternative parametrization via mean: see ŚDetailsŠ.
log, log.p logical; if TRUE, probabilities p are given as log(p).
lower.tail logical; if TRUE (default), probabilities are P[X = x], otherwise, P[X > x].
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