M1 Industrial Organisation
2016-2017 Final Examination

PLEASE READ THESE INSTRUCTIONS CAREFULLY:

- You have 1 hour.
- There are 14 pages (including this one). Make sure that they are all here.
- There are three questions, each worth the same amount of points. You should answer *one* question, and you should answer it *in this booklet.* If you answer more than one question we only mark one.
- Show your work. Unless otherwise indicated, partial credit may be given for partially correct work.
- Place your answer to each question in the space provided. Answers not provided in the correct space will not be marked.
- **Write answers neatly**. Illegible writing cannot be graded.
- **Good Luck!**

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1 Durable good monopoly

Question 1 : Explain in words the Coase conjecture for durable goods.

A monopolist sells a **durable good** over two periods, $t = 1, 2$. The marginal cost is zero. There is a mass 1 of consumers. Half of the consumers (L-types) value the good at 100 **per period**, while the other half (H-types) values it at 300 per period. There is no discounting ($\delta = 1$).

1.1 Leasing

Suppose that the firm leases the durable good. The timing is as follows: At $t = 1a$, the firm chooses a price r_1. At $t = 1b$, consumers choose whether to rent the good. At $t = 1c$, consumers who have rented the good return it. Period 2 follows the same pattern.

Question 2 What is the profit-maximizing (r_1, r_2)? What is the associated profit, consumer surplus, and total welfare?
Answer here:

1.2 Selling

Now we look at a situation where the firm sells the good. The timing is the following: At $t = 1a$ the firm chooses a price p_1. At $t = 1b$, consumers decide if they want to buy, and they keep the good for the two periods. At $t = 2a$, the firm chooses a price p_2. At $t = 2b$, consumers who haven't bought in period 1 decide whether to buy in period 2 at price p_2.

Consumers are **rational**, i.e. at any point in time they play their equilibrium strategy and expect the firm to do the same.

We start by looking at the subgame starting in period $2a$.

Question 3 Suppose that nobody bought the product in period 1. What is the optimal price p_2 for the firm? Same question if only the consumers with $v = 300$ bought the product in period 1.
Answer here:

Question 4 If the firm tries to sell to all the H-types in the first period, what is the maximal price it can charge in period 1? What is the profit over the two periods if the firm follows this strategy?
Answer here:

Question 5 If the firm tries to sell to both the L- and H-types in the first period, what is the maximal price it can charge in period 1? (Hint: assume that if the mass of consumers who buy in period 1 is 1, then $p_2 = 100$). What is the profit if the firm follows this strategy?

Question 6 Using your answers to questions 4 and 5, what is the equilibrium price structure (p_1, p_2)? Is the profit higher if the firm leases the product or if it sells it?

Question 7 Give one real-world example of a strategy used by firms to maintain high profits when selling durable goods.
Answer here:

2 Advertising

Two firms A and B are located at opposite ends of a unit-length Hotelling line. Consumers are uniformly distributed along the line and are interested in buying one product. Firm A can send two different types of advert, called 1 and 2 (e.g. 1 could be online advertising, and 2 could be advertising in magazines). If firm A advertises an amount $\lambda_1 \geq 0$ on 1, and an amount $\lambda_2 \geq 0$ on 2, its total cost of advertising is $\frac{(\lambda_1)^2+(\lambda_2)^2}{2}$. Marginal production cost for both firms is zero.

A consumer who buys from firm $i = A, B$ receives a payoff

$$V - p_i - d_i t\left(\lambda_1 + \lambda_2, \mu\right) ,$$

where p_i is the price charged by firm i, and d_i is the distance between the consumer and firm i. The transportation cost $t\left(\lambda_1 + \lambda_2, \mu\right)$ is a function, which depends on both firm A's total advertising $\lambda_1 + \lambda_2$, and an exogenous parameter μ; the transportation cost is strictly positive for all levels of advertising. We also assume that V is sufficiently high that the market is covered in equilibrium.

The timing of the game is as follows:

- At Stage 1, firm A chooses λ_1 and λ_2. They then become public knowledge.
- At Stage 2, firms A and B simultaneously choose their price.
- At Stage 3, consumers make their purchase decision.

Question 1 Write out the profit function for each firm at Stage 2, as a function of $p_A, p_B, \lambda_1, \lambda_2, \mu$.
Answer here:

Question 2 Solve for Nash Equilibrium prices and profits at Stage 2, as a function of $\lambda_1, \lambda_2, \mu$. You must show all your derivations. You do not need to check second order conditions.
Answer here:

Question 3 Prove that at Stage 1 firm A will choose $\lambda_1 = \lambda_2$. You should NOT assume anything about the function $t(\lambda_1 + \lambda_2, \mu)$ e.g. you should not assume it is differentiable. [*Hint: recall the lectures.*]
Answer here:

Henceforth we will assume that

$$t(\lambda_1 + \lambda_2, \mu) = 1 + \mu(\lambda_1 + \lambda_2) \ .$$

Question 4 Solve for firm A's optimal choice of λ_1 and λ_2 at the first stage of the game.
Answer here:

Let λ_1^0 and λ_2^0 denote the optimal λ_1 and λ_2 that you have solved for in question 4.

Question 5 Calculate the direct effect of a marginal change in λ_1 on firm A's final profit, starting from $\lambda_1 = \lambda_1^0$ and $\lambda_2 = \lambda_2^0$. You must show all your derivations.
Answer here:

Question 6 Calculate the strategic effect of a marginal change in λ_1 on firm A's final profit, starting from $\lambda_1 = \lambda_1^0$ and $\lambda_2 = \lambda_2^0$. You must show all your derivations.
Answer here:

Question 7 Explain why your answers to Questions 5 and 6 sum to zero.
Answer here:

3 Product differentiation and merger

Consider the circular city model. Consumers are located uniformly on a circle with a perimeter equal to 1. Density is unitary along the circle. There are four firms: each firm is located at one point of the circle and the distance between two adjacent firms' locations is equal to 1/4. More precisely, we assume that firm 1 is located at point 0, firm 2 at 1/4, firm 3 at 1/2 and firm 4 at 3/4. All the firms have the same marginal cost c.

Each firm sells an identical good and consumers have a unit demand. We assume that the gross surplus that each consumer obtains from the good (say,s) is high enough that the market is covered in any equilibrium: each consumer ends up buying a good. If a consumer located at x buys the good of firm i, she pays the price p_i and also incurs the transportation cost td_i where d_i is the distance between the consumer's location and firm i's location (all travel occurs along the circle!). For simplicity, we set $t = 1$. We study a game in which each firm simultaneously chooses its price.

1. Before the merger

Question 1 (a) Given the prices chosen by two adjacent firms 1 and 2, compute the location of the consumer who is indifferent between buying the good from firm 1 and buying it from firm 2. Compute also the location of the consumer who is indifferent between buying the good from firm 2 and buying it from firm 3.
Answer here:

Question 1 (b) From 1 (a), find the demand for firm 2 as a function of (p_1, p_2, p_3).
Answer here:

Question 1 (c) Write the expression of firm 2's profit π_2 as a function of (p_1, p_2, p_3).
Answer here:

Question 1 (d) From the FOC, find the price in the symmetric equilibrium. What is the profit per firm at the equilibrium?
Answer here:

2. Merger

Suppose now that two firms (firm 1 and firm 2) merge. We assume that (a) the locations of the firms are not affected by the merger, (b) the merging firms continue to choose prices (p_1, p_2). We keep studying the game in whch both the merging firms and the outsiders (i.e., firm 3 and firm 4) choose their price(s) simultaneously.

Question 2 (a) Given (p_1, p_2, p_3, p_4), write the expression of the profit of the merging firms $\pi_1 + \pi_2 \equiv \pi_I$ and write the expression of the profit of an outisder say π_3.
Answer here:

Question 2 (b) From the first order conditions, find the equilibrium prices $p_1 = p_2 = p_I$ and $p_3 = p_4 = p_O$.
Answer here:

Question 2 (c) (Incentive to merge) Does the merger increase or decrease the merging firms' joint profits? Give an economic intuition for your answer to the question.
Answer here:

Question 2 (d) Does the merger increase or decrease social welfare? Give an economic intuition for your answer to the question.
Answer here: