



# **Université Toulouse 1 Capitole Ecole d'économie de Toulouse**

Année universitaire 2016-201	17
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**Session 1** 

Semestre 2

Master 1 Economics, Econometrics & Statistics & Economics & Law

Epreuve: Industrial Organisation

Date de l'épreuve : 1 avril 2017

Durée de l'épreuve : 3h

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# M1 Industrial Organisation 2016-2017 Final Examination

Please	write	your	candidate	number	here:	
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#### PLEASE READ THESE INSTRUCTIONS CAREFULLY:

- This exam is for M1 Industrial Organisation.
- You have 3 hours. You should answer all questions, and you should answer them in this booklet.
- This exam has 4 questions, and is 16 pages long (including this one). Please check to make sure your copy has all 16 pages.
- The total number of points on the exam is 100. Each problem states the number of points it is worth. Allocate your time accordingly.
- Show your work. Unless otherwise indicated, partial credit may be given for partially correct work.
- Place your answer to each question in the space provided. Answers not provided in the correct space will not be marked.
- Write answers neatly. Illegible writing cannot be graded.
- Good Luck!

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# 1 Durable good monopoly (20 points)

A monopolist sells a durable good over two periods, t = 1, 2. The marginal cost is zero. There is a mass 1 of consumers. Half of the consumers (L-types) value the good at 100 per period, while the other half (H-types) values it at 300 per period. There is no discounting ( $\delta = 1$ ).

Question 1 Suppose that the firm leases the good at each period at prices  $r_1, r_2$ . What is the profit-maximizing  $(r_1, r_2)$ ? What is the associated profit, consumer surplus, and total welfare?

Answer here:

#### 1.1 Commitment

From now on we assume that the firm cannot lease the product, and must sell it.

Question 2 Give a reason why leasing may not be feasible. Answer here:

Suppose that, at t = 1, the firm can commit to prices  $p_1$  and  $p_2$ .

Question 3 Assuming that  $600 \ge p_1 \ge p_2$ , under which condition does a type H prefer to buy in the first period? Answer here:

Question 4 If the firm tries to sell to all the consumers (at t = 1 for the H-types, at t = 2 for the low types), what is the optimal price schedule  $(p_1, p_2)$  under commitment? Should the firm try to sell to all consumers in that case? Answer here:

	TO T	• • • • • • • • • • • • • • • • • • • •
1.2	No	commitment

Suppose now that the firm cannot commit to  $p_2$  at t = 1.

Question 5 What is the optimal strategy for the firm in that case? Compute equilibrium profit, consumer surplus and total welfare.

Answer here:

Question 6 Give one real-world example of a strategy used by firms to maintain high profits when selling durable goods. Answer here:

# 2 Collusion and competition in quantities (30 points)

Consider a market for a homogenous good, with an inverse demand P(Q) = 10 - Q, where Q is the total quantity. There are n firms,  $n \ge 1$ , who produce at constant marginal cost c = 2. Firms choose their quantities  $q_i$ .

Question 1 Suppose that n = 1. Show that the monopoly's optimal quantity is  $q^m = 4$ . Compute the monopolist's profit  $\pi^m$ . Compute consumer surplus, social welfare and the deadweight loss. Answer here:

Question 2 From now on, n = 2. What is the Nash equilibrium  $q^N$  of the static game? What is the associated profit  $\pi^N$ ?

Answer here:

Question 3 Now suppose that the game is repeated until period T. Each firm maximizes the present-discounted value of its profit

$$\sum_{t=0}^\infty \delta^t \pi_t^i$$

where  $\pi_t^i$  is firm i's profit at period t and  $\delta$  is the discount factor. What is the unique subgame-perfect equilibrium of this game? Is cooperation sustainable?

Answer here:

Question 4 From now on there is an infinite horizon:  $T=\infty$ . Firms try to collude to implement the symmetric monopoly outcome (where each firm produces  $\frac{q^m}{2}$  units at each period). We focus on subgame-perfect equilibria involving trigger-strategies.

Define each firm's trigger strategy.

Answer here:

Question 5 If firm 2 produces the collusive quantity at period t, show that the optimal deviation for firm 1 is to produce  $q^d = 3$ . What is the deviation profit for that period  $\pi^d$ ?

Answer here:

Question 6 able.	Use your previous answers	s to write down the inc	centive-compatibility	constraint for collusion	to be sustain
Answer here	e:				
result that col	Simplify the previous cond lusion is sustainable if and	lition to obtain a thres only if $\delta \geq \delta^*$ .	shold $\delta^*$ above which	collusion is sustainable.	Interpret the
Answer here	e:				
Question 8 Answer here	Briefly outline a model in	which price wars arise	on the equilibrium	collusive path.	

#### 3 Advertising (25 points)

Two firms A and B are located at opposite ends of a unit-length Hotelling line. Consumers are uniformly distributed along the line and are interested in buying one product. Firm A can send two different types of advert, called 1 and 2 (e.g. 1 could be online advertising, and 2 could be advertising in magazines). If firm A advertises an amount  $\lambda_1 \geq 0$  on 1, and an amount  $\lambda_2 \geq 0$  on 2, its total cost of advertising is  $\frac{(\lambda_1)^2 + (\lambda_2)^2}{2}$ . Marginal production cost for both firms is zero.

A consumer who buys from firm i = A, B receives a payoff

$$V-p_i-d_it\left(\lambda_1+\lambda_2,\mu\right)$$
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where  $p_i$  is the price charged by firm i, and  $d_i$  is the distance between the consumer and firm i. The transportation cost  $t(\lambda_1 + \lambda_2, \mu)$  is a function, which depends on both firm A's total advertising  $\lambda_1 + \lambda_2$ , and an exogenous parameter  $\mu$ ; the transportation cost is strictly positive for all levels of advertising. We also assume that V is sufficiently high that the market is covered in equilibrium.

The timing of the game is as follows:

- At Stage 1, firm A chooses  $\lambda_1$  and  $\lambda_2$ . They then become public knowledge.
- ullet At Stage 2, firms A and B simultaneously choose their price.
- At Stage 3, consumers make their purchase decision.

Question 1 Write out the profit function for each firm at Stage 2, as a function of  $p_A, p_B, \lambda_1, \lambda_2, \mu$ . Answer here:

Question 2 Solve for Nash Equilibrium prices and profits at Stage 2, as a function of  $\lambda_1, \lambda_2, \mu$ . You must show all your derivations. You do not need to check second order conditions. Answer here:

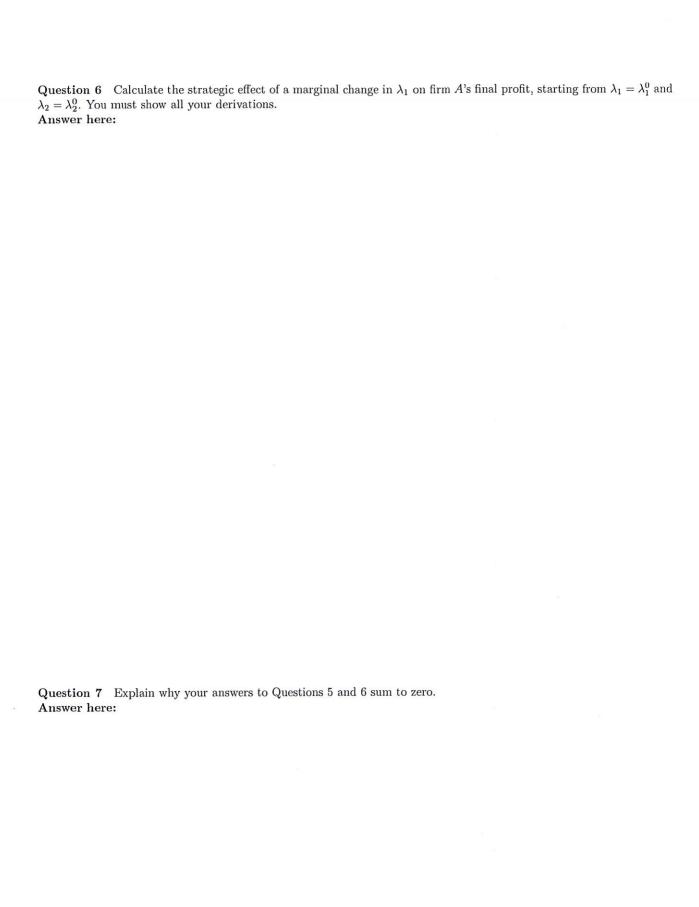
Question 3 Prove that at Stage 1 firm A will choose  $\lambda_1 = \lambda_2$ . You should NOT assume anything about the function  $t(\lambda_1 + \lambda_2, \mu)$  e.g. you should not assume it is differentiable. [Hint: recall the lectures.] Answer here:

Henceforth we will assume that

$$t\left(\lambda_1 + \lambda_2, \mu\right) = 1 + \mu\left(\lambda_1 + \lambda_2\right) .$$

Question 4 Solve for firm A's optimal choice of  $\lambda_1$  and  $\lambda_2$  at the first stage of the game. Answer here: Let  $\lambda_1^0$  and  $\lambda_2^0$  denote the optimal  $\lambda_1$  and  $\lambda_2$  that you have solved for in question 4.

Question 5 Calculate the direct effect of a marginal change in  $\lambda_1$  on firm A's final profit, starting from  $\lambda_1 = \lambda_1^0$  and  $\lambda_2 = \lambda_2^0$ . You must show all your derivations. Answer here:



## 4 Product differentiation and merger (25 points)

Consider the circular city model. Consumers are located uniformly on a circle with a perimeter equal to 1. Density is unitary along the circle. There are four firms: each firm is located at one point of the circle and the distance between two adjacent firms' locations is equal to 1/4. More precisely, we assume that firm 1 is located at point 0, firm 2 at 1/4, firm 3 at 1/2 and firm 4 at 3/4. All the firms have the same marginal cost c.

Each firm sells an identical good and consumers have a unit demand. We assume that the gross surplus that each consumer obtains from the good (say,s) is high enough that the market is covered in any equilibrium: each consumer ends up buying a good. If a consumer located at x buys the good of firm i, she pays the price  $p_i$  and also incurs the transportation cost  $td_i$  where  $d_i$  is the distance between the consumer and firm i (all travel occurs along the circle!). For simplicity, we set t = 1. We study a game in which each firm simultaneously chooses its price.

#### 1. Before the merger

Question 1 (a) Given the prices chosen by two adjacent firms  $(p_i, p_j)$ , compute the location of the consumer who is indifferent between buying the good from firm i and firm j.

Answer here:

Question 1 (b) From 1 (a), find the demand for firm 2 (for instance) as a function of  $(p_1, p_2, p_3)$ . Answer here:



#### 2. Merger

Suppose now that two firms (firm 1 and firm 2) merge. We assume that (a) the locations of the firms are not affected by the merging, (b) the merging firms continue to choose prices  $(p_1, p_2)$ . We keep studying the game in which both the merging firms and the outsiders (i.e., firm 3 and firm 4) choose their price(s) simultaneously.

Question 2 (a) Given  $(p_1, p_2, p_3, p_4)$ , write the expression of the profit of the merging firms  $\pi_1 + \pi_2 \equiv \pi_I$  and write the expression of the profit of an outisder say  $\pi_3$ .

Answer here:

Question 2 (b) From the first order conditions, find the equilibrium prices  $p_1 = p_2 = p_I$  and  $p_3 = p_4 = p_O$ . Answer here:

