Université Toulouse 1 Capitole
Ecole d'économie de Toulouse

Année universitaire 2016-2017

Session 1

Semestre 2

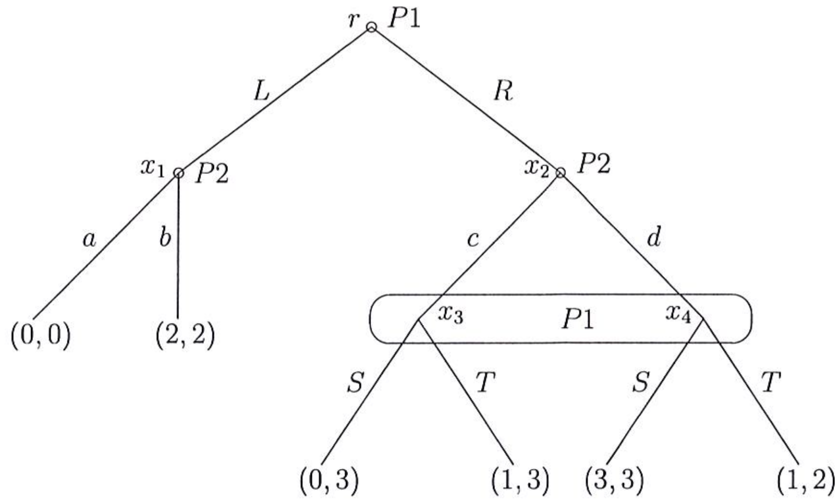
Master 1 Econometrics & Statistics
Epreuve : Decision Mathematics 2
Date de l'épreuve : 30 mars 2017
Durée de l'épreuve : 1h30

Liste des documents autorisés : aucun

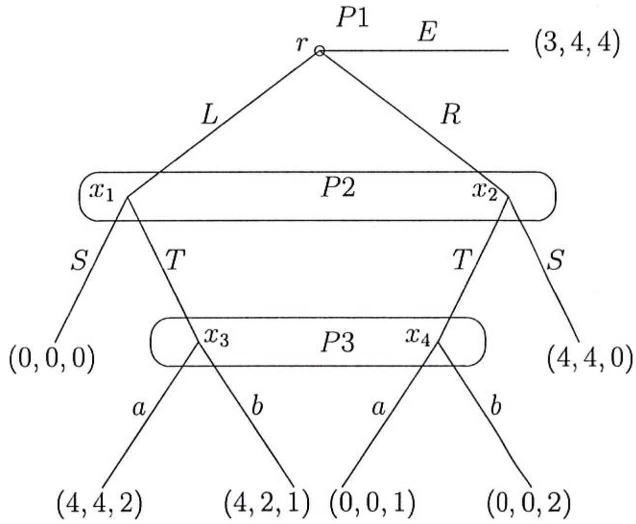
Liste des matériels autorisés : aucun

Nombre de pages (y compris page de garde): 3

Exercise 1. (3 pts) Compute the Nash equilibria and the subgame-perfect equilibria **in pure strategies**. Find a Bayesian-perfect equilibrium in pure strategies which is not a subgame-perfect equilibrium.



Exercise 2. (4pts) Compute the Nash equilibria, resp. the subgame perfect equilibria, resp. the Bayesian-perfect equilibria, resp. the sequential equilibria, **in pure strategies**.



Exercise 3. (8 pts) Consider the following game with three players where player 1 chooses a row, player 2 chooses a column, and player 3 chooses a matrix.

$$\begin{array}{c} \\ T \\ B \end{array} \begin{array}{c} \quad L \qquad\quad R \\ \begin{pmatrix} (0,0,0) & (1,1,2) \\ (1,2,1) & (2,1,1) \end{pmatrix} \\ W \end{array} \qquad \begin{array}{c} \quad L \qquad\quad R \\ \begin{pmatrix} (2,1,1) & (1,2,1) \\ (1,1,2) & (0,0,0) \end{pmatrix} \\ E \end{array}$$

1. Prove that there is no mixed Nash equilibrium in which player 3 is playing the pure strategy W.

2. With the same method as for question 1, it can be proved that there is no mixed Nash equilibrium in which some player is playing a pure strategy (no proof is required here). By using this result, compute all the mixed Nash equilibria.

3. Compute the correlated equilibrium distributions.

4. Find a correlated equilibrium distribution giving payoff 3/2 for player 1 and prove that it is the maximal possible payoff for player 1 in a correlated equilibrium.

Exercise 4. (5 pts) Let $G = (N, (A^i)_{i\in N}, (g^i)_{i\in N})$ be a finite game with $N = \{1, ..., n\}$. Let $\widetilde{G} = (N, \Delta(A^i)_{i\in N}, (\tilde{g}^i)_{i\in N})$ denote the mixed extension of G.

Definitions:

- A mixed action profile $\sigma = (\sigma^1, ..., \sigma^n) \in \prod_{i=1}^n \Delta(A^i)$ is completely mixed if

$$\forall i \in N, \forall a^i \in A^i,\ \sigma^i(a^i) > 0.$$

- A mixed action profile $\sigma = (\sigma^1, ..., \sigma^n) \in \prod_{i=1}^n \Delta(A^i)$ is a trembling-hand perfect equilibrium if there exists a sequence of completely mixed action profiles $(\sigma_k)_{k\geq 0}$ such that $\lim_{k\to\infty} \sigma_k = \sigma$ and

$$\forall k \geq 0,\ \sigma^i \text{ is a best reply against } \sigma_k^{-i}.$$

- A mixed action profile $\sigma = (\sigma^1, ..., \sigma^n) \in \prod_{i=1}^n \Delta(A^i)$ is an ε-perfect equilibrium if it is completely mixed and

$$\forall i \in N,\ \forall a^i \in A^i,\ \left(g^i(a^i, \sigma^{-i}) < \max_{b^i \in A^i} g^i(b^i, \sigma^{-i})\right) \Longrightarrow (\sigma^i(a^i) \leq \varepsilon).$$

Question:

Prove that if σ is a trembling-hand perfect equilibrium, then there exists a sequence $(\varepsilon_k, \sigma_k)_{k\in\mathbb{N}}$ such that: $\forall k$, $\varepsilon_k > 0$, σ_k is an ε_k-perfect equilibrium, $\varepsilon_k \longrightarrow_{k\to\infty} 0$ and $\sigma_k \longrightarrow_{k\to\infty} \sigma$.