Toulouse School of Economics
École d'économie de Toulouse

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Session 1

Semestre 2

Master 1 Economics, Econometrics & Statistics
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Final exam

Q1 True or False. No need to explain your answer.

1. In a second-price auction with (pure) common value, it is an equilibrium that every bidder bids exactly his valuation.

2. In a supply-function competition, an equilibrium outcome typically achieves a strictly lower expected social welfare than the team-efficient solution, because the condition for the optimal use of *private* information does not coincide with the condition for the socially efficient use of *private* information.

3. A sequential equilibrium requires that a belief system is consistent.

4. In a sequential equilibrium, every information set must be reached with a strictly positive probability.

5. Fix any Bayesian Nash equilibrium σ, and fix any belief system μ that is on-path consistent given σ. The combination (σ, μ) is a perfect Bayesian equilibrium.

Q2 Consider a private-value auction environment with n bidders. Each bidder i's valuation for the good $v_i \in [0, 1]$ follows a distribution with density $2v_i$. $v = (v_1, \ldots, v_n)$ is mutually independent.

1. Obtain the expected revenue for the seller in a second-price auction.

2. Consider the following auction rule (called an "all-pay" auction). Every bidder i simultaneously chooses $b_i \geq 0$; the highest bidder wins (in case of multiple highest bidders, each of them wins equally likely); and every bidder i pays b_i, *regardless of whether he wins or not.* That is, i's payoff is $v_i - b_i$ if he wins, and $-b_i$ if he loses. Assuming that there is an equilibrium where every bidder uses the same bidding strategy that

is strictly increasing, obtain the expected revenue for the seller in this auction.

3. Obtain a Bayesian Nash equilibrium of this game.

Q3 A seller of a used car knows its quality $\theta_1 \in \Theta_1 = \{10, 20, \ldots, 100\}$ as his private information. A buyer does not know it, but he believes that each θ_1 occurs equally likely. The timing of the game is as follows: at $t = 0$, Nature chooses θ_1; at $t = 1$, (without knowing θ_1) the buyer offers a price $a_2 \in \{11, 21, \ldots, 101\}$; at $t = 2$, (knowing a_2) the seller decides whether to sell it ($a_1 = 1$) or not ($a_1 = 0$).

For each θ_1, the seller's (opportunity) cost of selling the car is $v_1(\theta_1) = \theta_1$, and the buyer's valuation for the car is $v_2(\theta_1) = \theta_1 + 2$. Hence, the seller's payoff is $u_1(a_1, a_2, \theta_1) = a_1(a_2 - v_1(\theta_1))$ and the buyer's payoff is $u_2(a_1, a_2, \theta_1) = a_1(v_2(\theta_1) - a_2)$.

1. Prove that there is no pure PBE where the buyer offers $a_2 > 11$.

2. Obtain one PBE in this game.

3. Now change the timing slightly. Between $t = 0$ and $t = 1$, there is an additional stage $t = 0.5$, where the seller can send a *cheap-talk* message $m_1 \in M_1 = \Theta_1$. All the other parts of the game stay the same. Can the seller do better in some pure PBE? Explain.

4. Consider the same situation as in the last question with a cheap-talk message, but now assume that m_1 is a *hard-evidence* signaling. That is, the seller's payoff is

$$u_1(m_1, a_1, a_2, \theta_1) = \begin{cases} a_1(a_2 - v_1(\theta_1)) & \text{if } m_1 \leq \theta_1, \\ -1 & \text{if } m_1 > \theta_1, \end{cases}$$

while the buyer's payoff stays the same as before.

(a) Prove that there exists a separating PBE.

(b) Prove that there exists a pooling PBE.

(c) What is the refinement concept that eliminate this pooling PBE? Explain.