M1 Advanced Macro
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Retake Exam

You have 1 hour. Good luck!

1 The Romer model

This problem asks you to solve a variant of the Romer model where the ideas production function is given by

$$\dot{A}(t) = \eta Z(t),$$

where $Z(t)$ are units of the final good. The marginal costs of creating an idea are therefore given by $1/\eta$. Apart from this change, the model has the same structure as the Romer model presented in class.

To simplify the math, I do make, however, two more changes in functional forms. First, the final goods production function is given by

$$Y(t) = L^{1-\alpha}\left(\frac{1}{\alpha}\int_0^{A(t)} x_j(t)^\alpha \,\mathrm{d}j\right), \qquad 0 < \alpha < 1.$$

Second, the transformation rate of final goods into intermediate goods is given by $1/\alpha$. The marginal costs of intermediate-good firms is therefore α (i.e., one needs α units of the final good as input to produce one unit of a particular intermediate good variety).

1. Set up the final good producers' profit maximization problem and derive their demand for intermediate good variety j as a function of the intermediate good price p_j.

2. Set up the profit maximization problem of intermediate good producers (conditional on having a patent). Derive their profit-maximizing price, output, and profits.

3. Suppose that the interest-rate is constant. Compute the equilibrium price of patents.

4. Set up the profit maximization problem of research firms and derive the first-order condition for ideas production.

5. Use your answers to questions 3 and 4 to deduce the equilibrium interest-rate in the economy.

6. From the Euler equation of households it follows that along the balanced growth path it must hold that

$$g = \frac{1}{\sigma}(r - \rho).$$

Use this fact and your answers above to derive the rate of output growth, g, along the balanced growth path.

7. Is the equilibrium growth rate efficient? Discuss.

8. Suppose there is a competitive fringe of firms that can copy the innovation of a monopolist and produce at marginal costs $\psi\alpha$, with $\psi > 1$.

   (a) Compute the equilibrium price set by monopolists in the presence of the competitive fringe. Differentiate the case where $\psi \gtrless 1/\alpha$.

   (b) Use your result to recompute intermediate goods producer profits and deduce again the equilibrium price for patents, the optimal ideas production of research firms, and the resulting growth rate of the economy.

   (c) How does the growth rate vary with ψ? Discuss your results.

2 Club convergence

In the data we observe "club convergence": On the one hand, there is a convergence club consisting of most rich and middle-income countries that has converged to roughly the same long-run growth rate. On the other hand, many poor countries have been excluded from the club, having strictly lower (or zero) long-run growth rates.

1. Discuss inasmuch the Solow model can predict convergence to a common growth rate.

2. How about the endogenous growth models that you have learned in this course? Can you think of any mechanism that ensures convergence to a common growth rate?

3. Discuss inasmuch the models referred to in the previous two questions can account for the exclusion of poor countries from the club. Can you think of any mechanism that can account for the absence of convergence in non-club countries?