Toulouse School of Economics
École d'économie de Toulouse

UNIVERSITÉ TOULOUSE 1 CAPITOLE

Université Toulouse 1 Capitole
Ecole d'économie de Toulouse

Année universitaire 2016-2017

Session 1

Semestre 2

Master 1 Economics, Econometrics & Statistics
Epreuve : Advanced Macroeconomics
Date de l'épreuve : 28 mars 2017
Durée de l'épreuve : 1h30

Liste des documents autorisés : none

Liste des matériels autorisés : none

Nombre de pages (y compris page de garde) : 4

Robert Ulbricht
Toulouse School of Economics

Final Exam

You have 1.5 hours. Good luck!

1 Creative Destruction (6p.)

Consider a variant of the Schumpeterian growth model seen in the lecture. The production function for aggregate output is given by

$$Y(t) = \frac{1}{\alpha}\left(\int_0^1 q_j(t)^{1-\alpha} x_j(t)^\alpha \, \mathrm{d}j\right) L^{1-\alpha}.$$

Notice how quality now has an exponent $(1-\alpha)$, whereas before it entered linearly. Machines x_j are produced using the final good as input, where now one needs α units of the final good to produce one unit of a particularly machine variety (independently of the quality of the machine). All other features of the model remain unchanged. I.e., machine qualities evolve according to $q_j(t) = \lambda^{n_j(t)} q_j(0)$ where n_j measures the number of innovations made on machine line j. Quality improvements are made using the final good as input. In particular, if a potential entrant invests $Z_j(t)$ units of the final good, he makes improvements at a rate $\eta Z_j(t)/q_j(t)$ for some constant $\eta > 0$. The price of the final good is normalized to 1.

1. Assume that final good producers have access to only the highest quality of each machine currently available. Set up their profit maximization problem and derive their demand for machine variety j as a function of the machine price $p_j(t)$ and the current quality $q_j(t)$.

2. Set up the profit maximization problem of machine producers (conditional on owning the highest-quality patent). Derive the profit-maximizing price, output and profits for a given variety j. (You may assume that λ is sufficiently large so that we are in the radical innovation regime; i.e., monopolists face no relevant competition from lower-quality vintages).

3. Suppose that the interest rate r is constant and that innovations are made at a constant rate z. Compute the expected discounted monopoly profits $V(q_j(t))$ associated with being the highest-quality producer of a particular machine variety.

4. Set up the profit maximization problem of a potential entrant firm and derive the zero-profit condition for doing R&D. Use the condition to deduce the equilibrium interest rate as a function of the innovation rate z.

5. Use your answer to question 2 to derive an expression for aggregate output $Y(t)$. Use your answer to further deduce an expression for the growth rate of output, $g = \dot{Y}(t)/Y(t)$, as a function of the innovation rate z.

6. From the Euler equation of households it follows that along the balanced growth path it must hold that

$$g = \frac{1}{\sigma}(r - \rho).$$

Use this fact and your answers above to derive the rate of output growth along the balanced growth path.

2 Structural Change (7p.)

Consider the following model. There are two production technologies, given by

$$Y_1(t) = \frac{L_1(t)^{1-\alpha}}{1-\alpha} \quad \text{and} \quad Y_2(t) = A(t)L_2(t),$$

where L_1 and L_2 is the number of workers allocated to the two technologies, and $0 < \alpha < 1$. Labor market clearing requires $L_1(t) + L_2(t) = L(t)$, where the population $L(t)$ evolves exogenously according to

$$\dot{L}(t) = nL(t), \quad n > 0.$$

Productivity in Sector 2, $A(t)$, evolves according to

$$\dot{A}(t) = \phi L_2(t), \quad \phi > 0.$$

1. Give an economic interpretation for the law of motion governing productivity growth.

2. Consider the case where only technology 1 exists. Characterize the marginal product of labor (MPL) as a function of the population size. What happens to the MPL and output per worker in the long-run?

3. Now consider again the case where both technologies co-exist. Characterize the allocation of labor across the two technologies as a function of $L(t)$ and $A(t)$.

4. Suppose that the initial value of A satisfies $0 < A(0) < L(0)^{-\alpha}$. Are both technologies used at date 0? Does your answer change as time proceeds?

5. Define $u(t) = L_1(t)/L(t)$ as the fraction of the population that works in Sector 1, and define $g_u(t) = \dot{u}(t)/u(t)$ as the growth rate of u. For the case where both technologies are used, derive an expression for $g_u(t)$ as a function of n and the growth rate of A, $g_a(t) = \dot{A}(t)/A(t)$. What happens to $u(t)$ in the long-run?

6. Define $g_a(t) = \dot{A}(t)/A(t)$ as the growth rate of productivity in Sector 2.

   (a) Find the value of $g_a(0)$.

   (b) Use your answer to question 5 to derive an expression for $g_a(t)$ as a function of $L(t)$ and $A(t)$ as $t \to \infty$.

   (c) Use your previous answer to derive an expression for $\dot{g}_a(t)/g_a(t)$ (i.e., the growth in $g_a(t)$) as $t \to \infty$. Is there a stable steady state for $g_a(t)$?

   (d) Let $y(t) = (Y_1(t) + Y_2(t))/L(t)$ denote output per capita. Use your previous answers to derive the long-run growth rate of $y(t)$ as $t \to \infty$.

3 Club convergence (7p.)

In the data we observe "club convergence": On the one hand, there is convergence club consisting of most rich and middle-income countries that has converged to roughly the same long-run growth rate. On the other hand, many poor countries have been excluded from the club, having strictly lower (or zero) long-run growth rates.

Discuss how the models that you have learned in this course help understanding club convergence. Please keep your discussion clear. There are bonus points for a **precise** and **well-structured** argumentation! You may find it helpful to structure your discussion along the following points.

- Can you think of any mechanism that leads to convergence in growth rates?
- What could explain the absence of growth in per capita incomes in poor countries?
- How can the two answers be reconciled?