TSE M1 in Economics: Second Session Exam Probability Modeling.

Exercise 1
In a country, the probability p_n a family has n children is given by

$$p_n = a\frac{2^n}{n!} \text{ with } a > 0.$$

We assume that the probability to have a boy is the same than the probability to have a girl.

1. What is the value of a?
2. Compute the probability that at least a family has a boy.
3. We assume that a family has exactly one boy. What is the probability that the family has two children?

Hint: Introduce the event A_n : *the family has n children.*

Exercise 2:
Let (X, Y) be a Gaussian vector of variance $\begin{pmatrix} 1 & 1/4 \\ 1/4 & 1 \end{pmatrix}$

1. Explain why $(X + Y, X - Y)$ is a Gaussian vector?
2. Are $X + Y$ and $X - Y$ independent random variables?
3. Determine α such that $X - \alpha Y$ be independent of Y.
4. Deduce $\mathbb{E}((X - \frac{1}{4}Y)^2 Y^2)$.

Exercise 3:
Let (X, Y) be a Gaussian vector of variance $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

1. Explain why X and Y are independent random variables.
2. What is the distribution function of the random variable X^2?
3. What is the distribution function of the random variable $X^2 + Y^2$?