TSE
Toulouse School of Economics
École d'économie de Toulouse

UNIVERSITÉ TOULOUSE 1 CAPITOLE

Université Toulouse 1 Capitole
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Session 1

Semestre 1

Master 1 Econometrics, Statistics, Economics
Epreuve : Probability Modeling
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TSE M1 in Economics: Final Exam Probability Modeling.

Exercise 1
We throw two fair dices. We denote Y the result of the first dice and Z the result of the second dice. Then Y and Z are assumed to be identically distributed and independent. We set $X = Y + Z$.

1. Compute the distribution of X, that is $\mathbb{P}(X = k)$.
2. Compute the expectation of Y and Z.
3. Deduce the expectation of X.
4. Assume you observe $Z = 2$, what is the updated distribution of X?
5. Compute $\mathbb{E}(X|Z = 2)$.

Exercise 2
Let X be a gaussian random variable with mean zero and variance one. We define the random variables $Y = X^2$ and $Z = e^X$.

1. What is the probability distribution F_Y of Y?
2. What is the variance of Y?
3. What is the variance of Z?

Exercise 3
Let X be a random variable with density function $f(x) = xe^{-x}\mathbb{1}_{\{x\geq 0\}}$.

1. What is the probability distribution of X?
2. What is the probability distribution of $Y = X^2$?
3. Deduce the density of Y.
4. BONUS: Compute $\mathbb{E}(Y^2)$.

Exercise 4:
Let (X, Y) be a Gaussian vector of variance $\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$

1. Explain why $(X + Y, X - Y)$ is a Gaussian vector?
2. Are $X + Y$ and $X - Y$ independent random variables?
3. Determine α such that $X - \alpha Y$ be independent of Y.
4. Deduce $\mathbb{E}((X - \frac{1}{2}Y)^2Y^2)$.