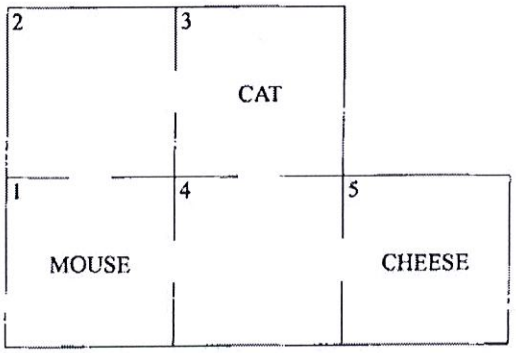
MARKOV CHAINS
EXAMINATION- SECOND SESSION
2 PAGES
YEAR : 2017 - M1 TSE
COURSE : O. FAUGERAS

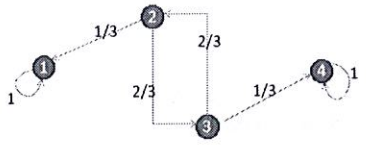
Closed book. Exercises are independent.
It is advised to provide careful reasoning and justifications in your answers. It will be taken a great care of them in the notation.

Exercise 1 A merry mouse moves in a maze. If it is at time n in a room with k horizontal or vertical adjacent rooms, it will be at time $n+1$ in one of the k adjacent rooms, choosing one at random, each with probability $1/k$. A fat lazy cat remains all the time in room 3, and a piece of cheese waits for the mouse in room 5. The mouse starts in room 1. See the following figure :



The cat is not completely lazy : If the mouse enters the room inhabited by the cat, the cat will eat it. What is the probability that the mouse ever gets to eat the cheese ?

Exercise 2
Let the homogeneous Markov chain $(X_n)_{n\in\mathbb{N}}$ be described by the following diagram :



1. What is the transition matrix ? Recall the conditions for a matrix to be a stochastic transition matrix.
2. What are the absorbing states ? Justify.
3. What is the probability to reach state 4, starting from state 2 ?
4. Starting from 2, what is the mean time to reach state 4 ?