TSE
Toulouse School of Economics
École d'économie de Toulouse

UNIVERSITÉ TOULOUSE 1 CAPITOLE

Université Toulouse 1 Capitole
Ecole d'économie de Toulouse

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Session 1

Semestre 1

Master 1 Econometrics, Statistics & Economics

Epreuve : Markov Chains & Applications

Date de l'épreuve : 13 décembre 2016

Durée de l'épreuve : 1h30

Liste des documents autorisés : Néant

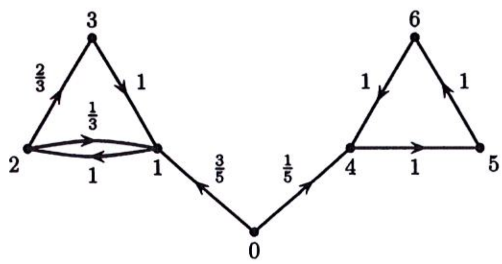
Liste des matériels autorisés : Calculatrice

Nombre de pages (y compris page de garde) : 3

Introduction to Stochastic Processes
EXAMINATION- first session
2 pages
Year : 2016 - M1 TSE
Course : O. Faugeras

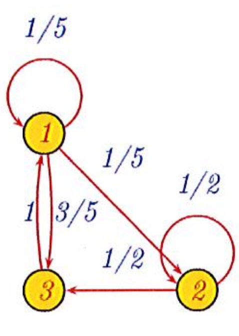
Closed book. Exercises are independent.
It is advised to provide careful reasoning and justifications in your answers. It will be taken a great care of them in the notation.

Exercise 1 Markov chains
Let the homogeneous Markov chain $(X_n)_{n\in\mathbb{N}}$ be described by the following graph :



1. Compute $P(X_1 = 0|X_0 = 0)$. Fill in the missing arrow on the graph.
2. What is the transition matrix ?
3. What are the communicating classes ? Which ones are closed ?
4. Starting from 0, what is the probability of hitting 6 ?
5. Starting from 1, what is the probability of hitting 3 ?
6. Starting from 1, how long does it take on average to hit 3 ?
7. Assume the chain starts from state 1. One now only focus on the states $\{1, 2, 3\}$ and one considers that the chain is only defined on these three states. What happens to the marginal distributions when $n \to \infty$? What is the invariant distribution ? What is the long-run proportion of time spent in 2 ?

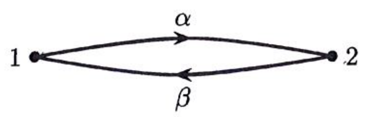
Exercise 2 Let the homogeneous Markov chain $(X_n)_{n\in\mathbb{N}}$ be described by the following graph :



The chain starts with the following initial distribution $\alpha_0 = (0.2 \quad 0.4 \quad 0.4)$. Compute

1. $P(X_3 = 1 | X_1 = 1)$,
2. $P(X_2 = 2)$,
3. $P(X_0 = 1, X_2 = 1)$,
4. $P(X_{100} = 1 | X_{99} = 3, X_{54} = 2)$,
5. What is the invariant distribution ? What happens to the chain if the initial distribution is equal to the invariant distribution ?
6. What is the average long run proportion of time spent in state 2 ?

Exercise 3 Let the Markov chain be described by the following diagram.



with $0 \leq \alpha, \beta < 1$. (1 is excluded for both probabilities)

1. What is the transition matrix ?
2. Starting from state 1, compute the probability to be in state 1 after n steps. Distinguish the cases between $\alpha + \beta = 0$ and $\alpha + \beta > 0$.
3. Find the limiting distribution as $n \to \infty$.