TSE

Toulouse School of Economics
École d'économie de Toulouse

UNIVERSITÉ TOULOUSE 1 CAPITOLE

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Ecole d'économie de Toulouse

Année universitaire 2016-2017

Session 1

Semestre 1

Master 1 Econometrics, Statistics, Economics & Economié Droit
Epreuve : Game Theory
Date de l'épreuve : 16 décembre 2016
Durée de l'épreuve : 2h

Liste des documents autorisés : None.

Liste des matériels autorisés : Casio calculator.

Nombre de pages (y compris page de garde) : 4 pages including this one.

Game Theory – Dec 2016 – 2 hours

There are three exercises. No documents. No computers. Nothing but paper and pens. Please provide clear and rigorous answers, with proofs. Your work must be written in English. The grading scale is indicative.

Exercise 1 (6 points): consider the following game, with three players 1, 2, and 3. In the first stage, player 3 chooses A or B. In the second stage, players 1 and 2 observe this choice, and then play a simultaneous game that depends on this choice: if player 3 has chosen A, they play the following game (in the matrix below player 1 is the row player, player 2 is the column player, and in each triple the first number is the payoff of player 1, the second number is the payoff of player 2 and the third number is the payoff of player 3):

	E	F
C	$3,0,2$	$1,1,2$
D	$2,1,2$	$2,2,2$

The A - game

And if player 3 has chosen B, players 1 and 2 play the following game (using the same conventions as above):

	E	F
C	$2,2,3$	$0,1,0$
D	$1,3,0$	$1,2,0$

The B - game

1) Give the set of pure strategies for every player. Find all subgame-perfect Nash equilibria in pure strategies. Give a simple reasoning (maximimum 10 lines, no computations) justifying that there are no other subgame-perfect Nash equilibria even if mixed strategies are allowed.

2) We now modify the game by adding more imperfect information. So in this question we assume that only player 2 observes the choice of player 3, while player 1 is left uninformed. Apart from this change, the game remains the same. Represent the game under extensive form (you do not have to indicate the payoffs in the tree.) Give the strategy set of every player. Find all subgame-perfect Nash equilibria in pure strategies in which player 3 plays A in the first stage.

Exercise 2 (5 points): Two players 1 and 2 want to go to the restaurant, either restaurant A or restaurant B. They would like having dinner together, but they may have different tastes concerning the choice of the restaurant. In this exercise, each player's taste is his own private information, as follows.

In the first stage, Nature draws independently two random variables, s_1 for player 1 and s_2 for player 2, which will be interpreted as the players' tastes for a particular restaurant. These two variables are identically and independently distributed, with two possible values a and b, each drawn with probability $1/2$. Player 1 learns the value of s_1 (but not the value of s_2), player 2 learns the value of s_2 (but not the value of s_1). To summarize, there are four states of Nature: $(a, a), (a, b), (b, a)$ and (b, b), each of them is drawn with probability $1/4$. For each state, the first element is the information received by player 1 and the second element is the information received by player 2.

In the second stage, players simultaneously choose a restaurant (either A or B).

Let $\beta \in]0, 1[$ be a parameter measuring the importance of being together, relative to going to his preferred restaurant. Payoffs are the sum of two terms: first, if the two players end up in the same restaurant, each of them earns β; otherwise they earn zero. In addition to this first term, if a player with taste a ends up in restaurant A he earns an additional payoff $(1 - \beta)$; same if a player with taste b ends up in restaurant B.

We focus on pure strategies.

1) Give the strategy sets.

2) Is there a Nash equilibrium in which both players always go to restaurant A? If yes, find it and prove it is indeed a NE; if no, prove it.

3) Is there a Nash equilibrium in which each player goes to his favorite restaurant? If yes, find it and prove it is a NE; if no, prove it.

Exercise 3 (9 points): there are three players: the Chief (C), the Supervisor (S), and the Agent (A). The Agent is lazy, and the Chief cannot observe whether the Agent exerts effort or not; so he has hired a Supervisor. The trouble is that the Supervisor is also lazy...

More precisely, the game is as follows. In the first stage of the game, the Chief chooses the reward $R \geq 0$. This choice is observed by other players. In the second stage of the game, the Agent decides to exert effort (E) or no effort (nE); and simultaneously the Supervisor decides to monitor (M) the Agent, or not (nM).

Action M costs m to the Supervisor. Action E costs e to the Agent. If the Supervisor chooses M and the Agent chooses nE, then the Supervisor can prove to the Chief that the Agent did not work. In this case, and in this case only, the Agent gets zero, and the Supervisor gets the reward R. In all other cases, the Agent is paid a wage w, and the Supervisor is paid zero. Finally, the Chief gets y if the Agent exerts effort, minus w if he pays the Agent's wage, minus R if he pays the Supervisor reward.

Assume that parameters m, e, w, y are strictly positive. Suppose also $y > w > e$. We are interested in Subgame-Perfect Nash Equilibria, and **we allow for mixed strategies.**

1) Give the (pure) strategy sets.

2) In this question we focus on the second-period simultaneous game between the Agent and the Supervisor, for R given. Put this game under matrix form (do not indicate the payoffs of the Chief). Solve this game in the following three cases:

Case a: when $R < m$;

Case b: when $R > m$: compute the best response functions, and find all Nash equilibria;

Case c: when $R = m$: use the analysis of point Case b to characterize the set of Nash equilibria.

Additionally, and in Case b only: suppose that this game is played twice, players observe what has been played at the end of each period, and there is no discounting ($\delta = 1$.) Give an intuitive argument showing that there exists a unique SPNE (maximum 10 lines, no computations).

3) In the case $w = m = 1$: consider the full game, including the first stage where the Chief chooses the reward. Does a SPNE exist?