Market Finance Exam

December 2015

1 Factor models (40 points)

1. State the efficient market hypothesis. Which evidence was used to support it?
2. Which evidence raised questions about the efficient market hypothesis? should the efficient market hypothesis be rejected? debate...
3. Briefly describe what a factor model is.
4. What is the CAPM model? Describe how the Fama-French 3-factor model disproves the CAPM.
5. Briefly describe the procedure for testing factor models.

2 Questions (20 points)

1. Define market completeness.
2. State the law of one price and the no-arbitrage condition. Can we have arbitrage if the law of one price apparently holds? Give a simple 3-states example.
3. Give the necessary condition for the existence of the pricing kernel m in $p = E(mx)$. Give the necessary conditions for it being positive. Why are we particularly interested in this case?
4. Under which conditions is the stochastic discount factor unique?
5. Show how the law of one price implies the existence of a factor model.

3 Problem (40 points)

Let the payoffs of the available assets be represented by the following matrix:

	x_1	x_2	x_3
s_1	1	4	14
s_2	2	1	21

1. Check that the market is complete.
2. Form Arrow Debreu securities as portfolios of these assets.
3. Let the price vector be given by $p = (3, 1, 5)$. Is the law of one price satisfied?
4. Same question for $p = (3, 1, 31)$. is the no arbitrage opportunity satisfied?
5. Same question for $p = (1, 3, 13)$. show both the law of one price and the no arbitrage opportunity are satisfied.
6. Assume equilibrium prices are $p = (1, 3, 13)$. Assuming, both states of nature are equally probable, compute a stochastic discount factor. Of which sign it is? Is it unique?
7. Compute the matrix of returns of a subset of two independent assets. Produce the vector of expected returns and their covariance matrix.
8. Find the mean-variance frontier for the given set of assets. Write down the statement of the problem, provide first order conditions and represent the solution graphically, labeling the axes and giving the precise location of the MVF.
9. Use the existence and unicity of the stochastic discount factor to define the risk-free asset as $\frac{1}{E(m)}$.
10. Find the tangency portfolio, and the set of efficient portfolios.
11. Bonus questions:
    (a) Assume, the representative investor has CRRA utility of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, at both date 0 and 1. Give expression for the SDF as a function of consumption at date 0 and 1.
    (b) Assume this investor wants to invest \$100 at time zero in order to consume at time 1 (she has already optimally consumed at time 0). How should she invest her \$100? Find the optimal portfolio of this consumer.