Toulouse School of Economics
École d'économie de Toulouse

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Session 1

Semestre 1

M1 Economics
M1 Economics & Law

Epreuve : Game Theory

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Liste des documents autorisés : None.

Liste des matériels autorisés : None.

Nombre de pages (y compris page de garde) : 4

Instructions. There are three exercises. No documents. No computers. Nothing but paper and pens. Give clear and rigorous justifications for your answers. Your work must be written in French, or in English -- not in both. The grading scale is indicative.

Exercise 1 (8 points): A Firm (F) proposes one unit of a good to a Consumer (C). The quality q of the good can take two values, low ($q = 1$) or high ($q = 2$). The consumer consumes either 0 or 1 unit. The price of the good is given : $p > 0$. If the consumer demands 0 unit, then both players get the same payoff, equal to 0. If the consumer consumes 1 unit of a good of quality q, then his payoff is $vq - p$, while the firm's payoff is $p - q^2$. We assume that the parameters v and p verify the following properties: $v > 3$, $p > 4$, and $v < p < 2v$.

The general question is whether there will be trade or not, and which quality will be traded, in different games that are defined below. We shall focus on pure-strategy, Subgame-Perfect Nash Equilibria (SPNE).

1) We define an efficient allocation as an allocation that maximizes the sum of the players' payoffs. Show that the efficient allocation is to have the consumer consuming one unit of the high quality good.

2) In this question the game is as follows. First, the firm chooses the quality of the good. Second, the consumer observes the quality of the good and decides to accept the trade (A) or to reject it (R).

Give the strategy sets of the firm and of the consumer. How many subgames in this game? Find the unique SPNE. Is it efficient?

3) In this question the game is as follows. First, the firm chooses the quality of the good. Second, the consumer does not observe the quality of the good, and decides to trade or not.

What is a strategy for the consumer? How many strategies for the consumer? How many subgames in this game? Find the unique SPNE. Is it efficient?

4) In this question the game is as follows. First, Nature draws a state: either O (with probability a) or NO (with probability $1 - a$), where a is a given parameter such that $0 < a < 1$. Second, the firm chooses the quality of the good, without knowing the state. Finally, the consumer learns the state. If the state is O, the consumer observes the quality of the good and decides to trade or not; if the state is NO, the consumer does not observe the quality of the good, and decides to trade or not.

Draw the corresponding tree of this extensive form game. Give the strategy set of the consumer. How many subgames in this game? What is the condition on the values of p and a for which there exists an efficient SPNE (recall that $v < p < 2v$)? Comment.

Exercise 2 (5 points): There are two firms A and B and two consumers a and b. Each firm can contact 0, 1, or 2 consumers, at a cost $c > 0$ per consumer. After this simultaneous stage, each consumer is either not contacted at all, and thus does not trade; either contacted by a single firm, and then he trades with this firm; either contacted by both firms, and then he trades with a firm chosen at random with equal probability (1/2).

A firm that contacts $n = 0, 1, 2$ consumers and trades with $k = 0, 1, 2$ consumers gets a payoff $kp - nc$, where $p > 0$ is a given parameter. Assume $p > 2c$. We focus on pure strategies.

1) Consider the simultaneous game G, the two firms being the two players. In this game, the two firms play simultaneously: each firm may contact consumer a, or consumer b, or both consumers, or none of them. Payoffs can then be computed from the rules given above.

Find the unique Nash equilibrium of G.

2) Now suppose that the game G is indefinitely repeated. Let $\delta < 1$ denote the discount factor. Consider the following strategy s_A for firm A:

"At the first period, I contact consumer a. At any future period, if in the past firm A has always contacted only consumer a and firm B has always contacted only consumer b, I contact consumer a; otherwise I contact both consumers."

Similarly for firm B:

"At the first period, I contact consumer b. At any future period, if in the past firm A has always contacted only consumer a and firm B has always contacted only consumer b, I contact consumer b; otherwise I contact both consumers."

Find the necessary and sufficient condition on the discount factor for (s_A, s_B) to form a SPNE of the repeated game.

Exercise 3 (7 points + bonus points): Consider the matching pennies game with two players A and B:

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

Figure 1: Matching pennies with complete information

We consider a variant of this game in which players have a personal taste for one action, in addition to the above payoffs. This taste is not known by the other player. We say that each player has two types, h or t. If the type of a player is h, then this player's payoff is increased by ε if he plays H. Similarly, if the type of a player is t, then this player's payoff is increased by ε if he plays T. We assume $0 < \varepsilon < 2$. ε is thus a perturbation of the game, and we would like to check that it does not perturb the equilibrium outcome of the matching pennies game too much when it is small.

So the timing of the game is as follows. At the first stage, Nature draws the types of the players: (h, h), (h, t), (t, h) or (h, t), each of them with probability $1/4$ (in each pair the first element defines the type of player A, and the second element defines the type of player B). At the second stage, player A observes his own type (h or t), he does not observe the type of player B, and he chooses H or T. At the third stage, player B observes his type (h or t), he does not observe the type of A, he does not observe the action chosen by player A, and he chooses H or T.

We focus on **pure strategies**.

1) Represent this extensive form game as a tree, without reporting the payoffs. Give the strategy sets of players.

2) The aim of this question is to characterize the set of Nash equilibria in pure strategies.

a) Suppose A plays H, whatever his type. Find the best response of player B.

b) Suppose A plays H if his type is h, and plays T if his type is t. Find the best response of player B.

c) Use symmetry arguments to show that there exists a unique pure strategy Nash equilibrium.

3) (Bonus question) We remind you that the unique Nash equilibrium of the matching pennies game with complete information is $((\frac{1}{2}H + \frac{1}{2}T), (\frac{1}{2}H + \frac{1}{2}T))$. Show that when $\varepsilon \to 0$ the outcome and payoffs of the (pure strategy) Nash equilibrium of the game with incomplete information converges to the outcome and payoffs of the (mixed strategy) Nash equilibrium of the game with complete information.