

**Université Toulouse 1 Capitole
Ecole d'économie de Toulouse**

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Session 1

Semestre 1

Master 1 Economics & Law

Epreuve : Econometrics

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Exercise 1 :

From a sample with size $N=72$ we consider the following model:

$$(M0) \quad y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 z_i^2 + u_i ; i= 1, \dots, 72$$

where $y=Ln(Y)$, $x=Ln(X)$, and $z=Ln(Z)$.

1. The following table gives the results for standard OLS estimation of (M0).
Comment these results (significance of coefficients, effects of regressors, main indicators, ...)

Dependent Variable: y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
x	-0.431301	0.083100	-5.190162	0.0000
z	1.746286	0.281536	6.202717	0.0000
z ²	-0.080676	0.043342	-1.861371	0.0670
C	-1.475663	0.393720	-3.748003	0.0004
R-squared	0.875062	Mean dependent var	2.301085	
Adjusted R-squared	0.869550	S.D. dependent var	1.152390	
S.E. of regression	0.416219	Akaike info criterion	1.138741	
Sum squared resid	11.78019	Schwarz criterion	1.265223	
Log likelihood	-36.99468	Hannan-Quinn criter.	1.189094	
F-statistic	158.7564	Durbin-Watson stat	2.445364	
Prob(F-statistic)	0.000000			

2. In the following table, u is the residual from the previous regression.

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	9.899942	Prob. F(1,70)	0.0024
Obs*R-squared	8.921106	Prob. Chi-Square(1)	0.0028
Scaled explained SS	11.07997	Prob. Chi-Square(1)	0.0009

Test Equation:
Dependent Variable: u²

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.470531	0.102216	4.603317	0.0000
z	-0.110093	0.034990	-3.146417	0.0024

- a. Comment these results (why we do that, conclusions, consequences for results in question 1), ...)
- b. Give the procedure to apply to continue this study, if necessary.

Exercise 2

We consider the model: $Y_i = \beta_0 + \beta_1 X_i + \beta_2 W_i + u_i ; i = 1, \dots, N$, where u iid $N(0, \sigma^2)$. We suspect W to be an endogenous regressor, and we have data on 2 instrumental variables (IV) denoted $Z1$, and $Z2$.

1. Give the 3 main sources of an endogeneity problem.
2. What are the conditions for $Z1$ and $Z2$ to be valid IV?
3. The following tables give the estimation results of coefficients by OLS and by 2SLS using a sample with size $N=521$. Comment these results and explain the procedure used in 2SLS method.

Table 1: OLS

Dependent Variable: Y				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
W	0.081308	0.007595	10.70525	0.0000
X	0.020559	0.003840	5.353444	0.0000
C	5.494579	0.129670	42.37363	0.0000
R2	0.14688			
SSR	100.318			

Table 2: 2SLS

Dependent Variable: W				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
X	-0.189646	0.019840	-9.558659	0.0000
Z1	0.103125	0.035782	2.882002	0.0041
Z2	0.182623	0.030732	5.942432	0.0000
C	12.87152	0.448661	28.68877	0.0000
R2	0.3092			
SSR	1798.943			

Dependent Variable: Y				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
W	0.139216	0.022957	6.064101	0.0000
X	0.035079	0.006765	5.185507	0.0000
C	4.544295	0.379093	11.98730	0.0000
R2	0.0321			
SSR	86.01275			

4. When we regress W on X only, we obtain: $SSR=2603.341$. What is your conclusion about “weakness” of IV?
5. Comment the following OLS estimation results, where \hat{V} is the residual from 1st stage of 2SLS.

Dependent Variable: Y				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
W	0.139216	0.021536	6.464195	0.0000
X	0.035079	0.006346	5.527634	0.0000
\hat{V}	-0.076267	0.023346	-3.266786	0.0012
C	4.544295	0.355629	12.77819	0.0000
R2	0.149903			
SSR	75.54879			

6. Describe the procedure to apply in order to test overidentifying restrictions in this model.

Exercise 3

We are interested in a model to explain the decision by married women to work or no. We use a random sample of N=4148 married women (with husband working), for which we observe a variable for working decision (Y = 1 if working, 0 otherwise) and the following variables: AGE, years of education (EDUC), a binary variable KID (=1 if have child under 6 years, 0 otherwise), and husband's wage (HUSWAGE). We estimate by OLS the regression of Y on previously defined regressors and we obtain:

Dependent Variable: Y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AGE	-0.006250	0.000857	-7.290763	0.0000
EDUC	0.042797	0.002923	14.64081	0.0000
KID	-0.212667	0.017948	-11.84940	0.0000
HUSWAGE	-0.013577	0.002175	-6.243157	0.0000
C	0.449012	0.051536	8.712584	0.0000
R-squared	0.081308			
Adjusted R-squared	0.080421			
S.E. of regression	0.465387			
Sum squared resid	897.3136			
Log likelihood	-2710.512			
F-statistic	91.66866			
Prob(F-statistic)	0.000000			

1. Comment these results and give advantages and disadvantages of such a model.
2. We consider now a *Logit model* for the previous relation.

Reminder: if Y has a standard logistic distribution, the density is $f(y) = \exp(-y)/(1+\exp(-y))^2$ and cdf is $F(y)=1/(1+\exp(-y))$.

- a. By denoting X the matrix of regressors and constant, and β the vector of coefficients, describe what is a Logit model as derived from an underlying latent model.
- b. Estimation of the Logit model gives:

Dependent Variable: Y

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AGE	-0.029978	0.004086	-7.336610	0.0000
EDUC	0.202830	0.014859	13.65065	0.0000
KID	-0.987356	0.084822	-11.64033	0.0000
HUSWAGE	-0.063120	0.010013	-6.304112	0.0000
C	-0.280045	0.245707	-1.139755	0.2544
McFadden R-squared	0.064200	Mean dependent var	0.620540	
S.D. dependent var	0.485311	S.E. of regression	0.465151	
Akaike info criterion	1.244778	Sum squared resid	896.4011	
Schwarz criterion	1.252409	Log likelihood	-2576.669	
Hannan-Quinn criter.	1.247478	Deviance	5153.339	
Restr. deviance	5506.878	Restr. log likelihood	-2753.439	
LR statistic	353.5390	Avg. log likelihood	-0.621184	
Prob(LR statistic)	0.000000			
Obs with Dep=0	1574	Total obs	4148	
Obs with Dep=1	2574			

For a woman without kids, with 14 years of schooling, a value of husband's wage equal to 10 and 35 years old, compute (consider only 3 decimal places without rounding for coefficients):

- (i) the probability of working.
- (ii) the effect of *age* on the probability of working.

3. Finally, we estimate a **Tobit** model explaining the number of working hours (HOURS) in terms of AGE, EDUC, KID and HUSWAGE, using the same sample as previously, and we obtain:

Dependent Variable: HOURS				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
AGE	-0.383448	0.054261	-7.066788	0.0000
EDUC	2.813799	0.191892	14.66343	0.0000
KID	-14.57760	1.144721	-12.73463	0.0000
HUSWAGE	-0.996712	0.139344	-7.152878	0.0000
C	3.605370	3.307096	1.090192	0.2756
Error Distribution				
SCALE:C(6)	27.69650	0.429903	64.42499	0.0000
Mean dependent var	22.17310	S.D. dependent var	19.27526	
S.E. of regression	18.49190	Akaike info criterion	6.538368	
Sum squared resid	1416700.	Schwarz criterion	6.547525	
Log likelihood	-13554.58	Hannan-Quinn criter.	6.541608	
Avg. log likelihood	-3.267738			
Left censored obs	1574	Right censored obs	0	
Uncensored obs	2574	Total obs	4148	

Remember : « SCALE:C(6) » gives estimate of σ , using course's notation.

- Give the justification for using a Tobit in this case.
- Estimate the difference in expected hours for a woman with kids and a woman without kids, both with 14 years of schooling, a value of husband's wage equal to 10 and 35 years old. (consider only 3 decimal places without rounding for coefficients)

TABLES STATISTIQUES

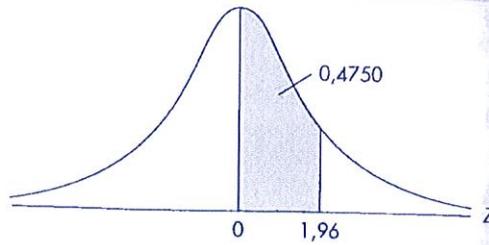
$$N(0,1)$$

Table D.1 Surfaces sous la distribution normale standardisée

Exemple

$$\Pr(0 \leq Z \leq 1,96) = 0,4750$$

$$\Pr(Z \geq 1,96) = 0,5 - 0,4750 = 0,025$$

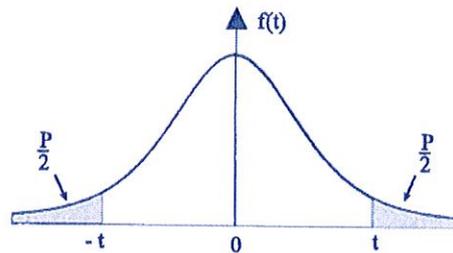


Z	,00	,01	,02	,03	,04	,05	,06	,07	,08	,09
0,0	,0000	,0040	,0080	,0120	,0160	,0199	,0239	,0279	,0319	,0359
0,1	,0398	,0438	,0478	,0517	,0557	,0596	,0636	,0675	,0714	,0753
0,2	,0793	,0832	,0871	,0910	,0948	,0987	,1026	,1064	,1103	,1141
0,3	,1179	,1217	,1255	,1293	,1331	,1368	,1406	,1443	,1480	,1517
0,4	,1554	,1591	,1628	,1664	,1700	,1736	,1772	,1808	,1844	,1879
0,5	,1915	,1950	,1985	,2019	,2054	,2088	,2123	,2157	,2190	,2224
0,6	,2257	,2291	,2324	,2357	,2389	,2422	,2454	,2486	,2517	,2549
0,7	,2580	,2611	,2642	,2673	,2704	,2734	,2764	,2794	,2823	,2852
0,8	,2881	,2910	,2939	,2967	,2995	,3023	,3051	,3078	,3106	,3133
0,9	,3159	,3186	,3212	,3238	,3264	,3289	,3315	,3340	,3365	,3389
1,0	,3413	,3438	,3461	,3485	,3508	,3531	,3554	,3577	,3599	,3621
1,1	,3643	,3665	,3686	,3708	,3729	,3749	,3770	,3790	,3810	,3830
1,2	,3849	,3869	,3888	,3907	,3925	,3944	,3962	,3980	,3997	,4015
1,3	,4032	,4049	,4066	,4082	,4099	,4115	,4131	,4147	,4162	,4177
1,4	,4192	,4207	,4222	,4236	,4251	,4265	,4279	,4292	,4306	,4319
1,5	,4332	,4345	,4357	,4370	,4382	,4394	,4406	,4418	,4429	,4441
1,6	,4452	,4463	,4474	,4484	,4495	,4505	,4515	,4525	,4535	,4545
1,7	,4454	,4564	,4573	,4582	,4591	,4599	,4608	,4616	,4625	,4633
1,8	,4641	,4649	,4656	,4664	,4671	,4678	,4686	,4693	,4699	,4706
1,9	,4713	,4719	,4726	,4732	,4738	,4744	,4750	,4756	,4761	,4767
2,0	,4772	,4778	,4783	,4788	,4793	,4798	,4803	,4808	,4812	,4817
2,1	,4821	,4826	,4830	,4834	,4838	,4842	,4846	,4850	,4854	,4857
2,2	,4861	,4864	,4868	,4871	,4875	,4878	,4881	,4884	,4887	,4890
2,3	,4893	,4896	,4898	,4901	,4904	,4906	,4909	,4911	,4913	,4916
2,4	,4918	,4920	,4922	,4925	,4927	,4929	,4931	,4932	,4934	,4936
2,5	,4938	,4940	,4941	,4943	,4945	,4946	,4948	,4949	,4951	,4952
2,6	,4953	,4955	,4956	,4957	,4959	,4960	,4961	,4962	,4963	,4964
2,7	,4965	,4966	,4967	,4968	,4969	,4970	,4971	,4972	,4973	,4974
2,8	,4974	,4975	,4976	,4977	,4977	,4978	,4979	,4979	,4980	,4981
2,9	,4981	,4982	,4982	,4983	,4984	,4984	,4985	,4985	,4986	,4986
3,0	,4987	,4987	,4987	,4988	,4988	,4989	,4989	,4989	,4990	,4990

Note : Cette table donne la surface sous la partie droite de la distribution (c'est-à-dire $Z \geq 0$). Mais puisque la distribution normale est symétrique autour de $Z = 0$, l'aire sur la partie gauche est identique à sa correspondante de la partie droite. Par exemple, $P(-1,96 \leq Z \leq 0) = 0,4750$. D'où, $P(-1,96 \leq Z \leq 1,96) = 2(0,4750) = 0,95$.

Table de la loi de Student

Valeurs de T ayant la probabilité P d'être dépassées en valeur absolue



Exemples:

pour $\alpha = 2\alpha$

$$P_2(T > 2,086) = 0,025$$

$$P_2(|T| > 2,086) = 0,05$$

$$P_2(T > 1,325) = 0,20$$

ν	$P = 0,90$	0,80	0,70	0,60	0,50	0,40	0,30	0,20	0,10	0,05	0,02	0,01
1	0,158	0,325	0,510	0,727	1,000	1,376	1,963	3,078	6,314	12,706	31,821	63,657
2	0,142	0,289	0,445	0,617	0,816	1,061	1,386	1,886	2,920	4,303	6,965	9,925
3	0,137	0,277	0,424	0,584	0,765	0,978	1,250	1,638	2,353	3,182	4,541	5,841
4	0,134	0,271	0,414	0,569	0,741	0,941	1,190	1,533	2,132	2,776	3,747	4,604
5	0,132	0,267	0,408	0,559	0,727	0,920	1,156	1,476	2,015	2,571	3,365	4,032
6	0,131	0,265	0,404	0,553	0,718	0,906	1,134	1,440	1,943	2,447	3,143	3,707
7	0,130	0,263	0,402	0,549	0,711	0,896	1,119	1,415	1,895	2,365	2,998	3,499
8	0,130	0,262	0,399	0,546	0,706	0,889	1,108	1,397	1,860	2,306	2,896	3,355
9	0,129	0,261	0,398	0,543	0,703	0,883	1,100	1,383	1,833	2,262	2,821	3,250
10	0,129	0,260	0,397	0,542	0,700	0,879	1,093	1,372	1,812	2,228	2,764	3,169
11	0,129	0,260	0,396	0,540	0,697	0,876	1,088	1,363	1,796	2,201	2,718	3,106
12	0,128	0,260	0,395	0,539	0,695	0,873	1,083	1,356	1,782	2,179	2,681	3,055
13	0,128	0,259	0,394	0,538	0,694	0,870	1,079	1,350	1,771	2,160	2,650	3,012
14	0,128	0,258	0,393	0,537	0,692	0,868	1,076	1,345	1,761	2,145	2,624	2,977
15	0,128	0,258	0,393	0,536	0,691	0,866	1,074	1,341	1,753	2,131	2,602	2,947
16	0,128	0,258	0,392	0,535	0,690	0,865	1,071	1,337	1,746	2,120	2,583	2,921
17	0,128	0,257	0,392	0,534	0,689	0,863	1,069	1,333	1,740	2,110	2,567	2,898
18	0,127	0,257	0,392	0,534	0,688	0,862	1,067	1,330	1,734	2,101	2,552	2,878
19	0,127	0,257	0,391	0,533	0,688	0,861	1,066	1,328	1,729	2,093	2,539	2,861
20	0,127	0,257	0,391	0,533	0,687	0,860	1,064	1,325	1,725	2,086	2,528	2,845
21	0,127	0,257	0,391	0,532	0,686	0,859	1,063	1,323	1,721	2,080	2,518	2,831
22	0,127	0,256	0,390	0,532	0,686	0,858	1,061	1,321	1,717	2,074	2,508	2,819
23	0,127	0,256	0,390	0,532	0,685	0,858	1,060	1,319	1,714	2,069	2,500	2,807
24	0,127	0,256	0,390	0,531	0,685	0,857	1,059	1,318	1,711	2,064	2,492	2,797
25	0,127	0,256	0,390	0,531	0,684	0,856	1,058	1,316	1,708	2,060	2,485	2,787
26	0,127	0,256	0,390	0,531	0,684	0,856	1,058	1,315	1,706	2,056	2,479	2,779
27	0,127	0,256	0,389	0,531	0,684	0,855	1,057	1,314	1,703	2,052	2,473	2,771
28	0,127	0,256	0,389	0,530	0,683	0,855	1,056	1,313	1,701	2,048	2,467	2,763
29	0,127	0,256	0,389	0,530	0,683	0,854	1,055	1,311	1,699	2,045	2,462	2,756
30	0,127	0,256	0,389	0,530	0,683	0,854	1,055	1,310	1,697	2,042	2,457	2,750
∞	0,126	0,253	0,385	0,524	0,674	0,842	1,036	1,282	1,645	1,96	2,326	2,576

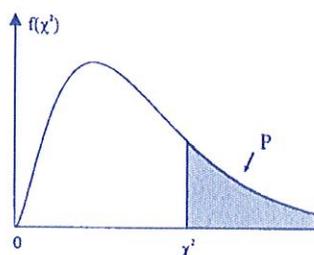
Nota. ν est le nombre de degrés de liberté.

Le quantile d'ordre $1 - \frac{\alpha}{2}$ se lit dans la colonne $P = \alpha$.

Le quantile d'ordre $1 - \alpha$ se lit dans la colonne $P = 2\alpha$.

Table de la loi du Khi-deux

Valeurs de χ^2 ayant la probabilité P d'être dépassées



Exemples
 pour $\nu = 20$:
 $P_2(\chi^2 > 10,85) = 0,95$
 $P_2(\chi^2 > 31,41) = 0,05$

ν	P = 0,995	0,99	0,975	0,95	0,90	0,10	0,05	0,025	0,01	0,005
1	0,00004	0,0002	0,001	0,0039	0,0158	2,706	3,841	5,024	6,635	7,879
2	0,010	0,020	0,051	0,103	0,211	4,605	5,991	7,378	9,210	10,597
3	0,072	0,115	0,216	0,352	0,584	6,251	7,815	9,348	11,345	12,838
4	0,207	0,297	0,484	0,711	1,064	7,779	9,488	11,143	13,277	14,860
5	0,412	0,554	0,831	1,145	1,610	9,236	11,070	12,833	15,086	16,750
6	0,676	0,872	1,237	1,635	2,204	10,645	12,592	14,449	16,812	18,548
7	0,989	1,239	1,690	2,167	2,833	12,017	14,067	16,013	18,475	20,278
8	1,344	1,646	2,180	2,733	3,490	13,362	15,507	17,535	20,090	21,955
9	1,735	2,088	2,700	3,325	4,168	14,684	16,919	19,023	21,666	23,589
10	2,156	2,558	3,247	3,940	4,865	15,987	18,307	20,483	23,209	25,188
11	2,603	3,053	3,816	4,575	5,578	17,275	19,675	21,920	24,725	26,757
12	3,074	3,571	4,404	5,226	6,304	18,549	21,026	23,337	26,217	28,300
13	3,565	4,107	5,009	5,892	7,042	19,812	22,362	24,736	27,688	29,819
14	4,075	4,660	5,629	6,571	7,790	21,064	23,685	26,119	29,141	31,319
15	4,601	5,229	6,262	7,261	8,547	22,307	24,996	27,488	30,578	32,801
16	5,142	5,812	6,908	7,962	9,312	23,542	26,296	28,845	32,000	34,267
17	5,697	6,408	7,564	8,672	10,085	24,769	27,587	30,191	33,409	35,718
18	6,265	7,015	8,231	9,39	10,865	25,989	28,869	31,526	34,805	37,156
19	6,844	7,633	8,907	10,117	11,651	27,204	30,144	32,852	36,191	38,582
20	7,434	8,260	9,591	10,851	12,443	28,412	31,410	34,170	37,566	39,997
21	8,034	8,897	10,283	11,591	13,240	29,615	32,671	35,479	38,932	41,401
22	8,643	9,542	10,982	12,338	14,041	30,813	33,924	36,781	40,289	42,796
23	9,260	10,196	11,689	13,091	14,848	32,007	35,172	38,076	41,638	44,181
24	9,886	10,856	12,401	13,848	15,659	33,196	36,415	39,364	42,980	45,559
25	10,520	11,524	13,120	14,611	16,473	34,382	37,652	40,646	44,314	46,928
26	11,160	12,198	13,844	15,379	17,292	35,563	38,885	41,923	45,642	48,290
27	11,808	12,879	14,573	16,151	18,114	36,741	40,113	43,195	46,963	49,645
28	12,461	13,565	15,308	16,928	18,939	37,916	41,337	44,461	48,278	50,993
29	13,121	14,256	16,047	17,708	19,768	39,087	42,557	45,722	49,588	52,336
30	13,787	14,953	16,791	18,493	20,599	40,256	43,773	46,979	50,892	53,672

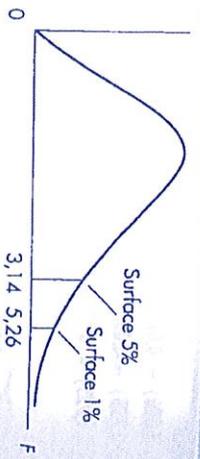
Nota. ν est le nombre de degrés de liberté.

Pour $\nu > 30$, on peut admettre que la quantité $\sqrt{2\chi^2} - \sqrt{2\nu - 1}$ suit la loi normale centrée réduite.

Table D.3 Valeurs critiques supérieures de la distribution F

Exemple
 $Pr(F > 1,59) = 0,25$
 $Pr(F > 2,42) = 0,10$
 $Pr(F > 3,14) = 0,05$
 $Pr(F > 5,26) = 0,01$

pour $dl_1 = 10$
 et $M_2 = 9$



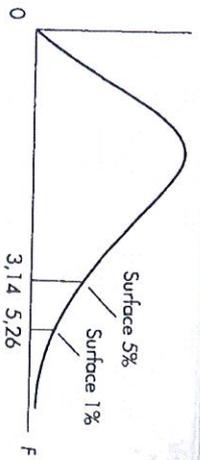
dl pour le dénominateur M_2	Pr	Degrés de liberté pour le numérateur M_1											
		1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5,83	7,50	8,20	8,58	8,82	8,98	9,10	9,19	9,26	9,32	9,36	9,41
	.10	39,9	49,5	53,6	55,8	57,2	58,2	58,9	59,4	59,9	60,2	60,5	60,7
2	.25	2,57	3,00	3,15	3,23	3,28	3,31	3,34	3,35	3,37	3,38	3,39	3,39
	.10	8,53	9,00	9,16	9,24	9,29	9,33	9,35	9,37	9,38	9,39	9,40	9,41
3	.25	2,02	2,28	2,36	2,39	2,41	2,42	2,43	2,44	2,44	2,45	2,45	2,45
	.10	5,54	5,46	5,39	5,34	5,31	5,28	5,27	5,25	5,24	5,23	5,22	5,22
4	.25	1,81	2,00	2,05	2,06	2,07	2,08	2,08	2,08	2,08	2,08	2,08	2,08
	.10	4,54	4,32	4,19	4,11	4,05	4,01	3,98	3,95	3,94	3,92	3,91	3,90
5	.25	1,69	1,85	1,88	1,89	1,89	1,89	1,89	1,89	1,89	1,89	1,89	1,89
	.10	4,06	3,78	3,62	3,52	3,45	3,40	3,37	3,34	3,32	3,30	3,28	3,27
6	.25	1,62	1,76	1,78	1,79	1,79	1,78	1,78	1,78	1,77	1,77	1,77	1,77
	.10	3,78	3,46	3,29	3,18	3,11	3,05	3,01	2,98	2,96	2,94	2,92	2,90
7	.25	1,57	1,70	1,72	1,72	1,71	1,71	1,70	1,70	1,69	1,69	1,68	1,68
	.10	3,59	3,26	3,07	2,96	2,88	2,83	2,78	2,75	2,72	2,70	2,68	2,67
8	.25	1,54	1,66	1,67	1,66	1,66	1,65	1,64	1,63	1,63	1,63	1,63	1,62
	.10	3,46	3,11	2,92	2,81	2,73	2,67	2,62	2,59	2,56	2,54	2,52	2,50
9	.25	1,51	1,62	1,63	1,63	1,62	1,61	1,60	1,59	1,59	1,58	1,58	1,58
	.10	3,38	3,01	2,81	2,69	2,61	2,55	2,51	2,47	2,44	2,42	2,40	2,38
10	.25	1,50	1,61	1,62	1,61	1,61	1,60	1,60	1,59	1,59	1,58	1,58	1,58
	.10	3,36	2,98	2,78	2,66	2,58	2,52	2,48	2,44	2,41	2,39	2,37	2,35
11	.25	1,49	1,60	1,61	1,60	1,60	1,59	1,58	1,58	1,57	1,57	1,57	1,57
	.10	3,34	2,96	2,76	2,64	2,56	2,50	2,46	2,42	2,39	2,37	2,35	2,33
12	.25	1,48	1,59	1,60	1,59	1,59	1,58	1,58	1,57	1,57	1,56	1,56	1,56
	.10	3,32	2,94	2,74	2,62	2,54	2,48	2,44	2,40	2,37	2,35	2,33	2,31

Source : Tiré de E. S. Pearson et H. O. Hartley, éd., *Biometrika Tables for Statisticians*, vol. 1, 3^e éd., table 18, Cambridge University Press, New York, 1966. Reproduit avec l'autorisation des auteurs et des administrateurs de Biometrika.

Table D.3 Valeurs critiques supérieures de la distribution F (suite)

Exemple
 $Pr(F > 1,59) = 0,25$
 $Pr(F > 2,42) = 0,10$
 $Pr(F > 3,14) = 0,05$
 $Pr(F > 5,26) = 0,01$

pour $dl_1 = 10$
 et $M_2 = 9$



dl pour le dénominateur M_2	Pr	Degrés de liberté pour le numérateur M_1																	
		15	20	24	30	40	50	60	100	120	200	500	∞						
1	.25	9,49	9,58	9,63	9,67	9,71	9,74	9,76	9,78	9,80	9,82	9,84	9,85						
	.10	61,2	61,7	62,0	62,3	62,5	62,7	62,8	63,0	63,1	63,2	63,3	63,3						
2	.25	3,41	3,43	3,43	3,44	3,45	3,45	3,46	3,47	3,47	3,48	3,48	3,48						
	.10	9,42	9,44	9,45	9,46	9,47	9,47	9,47	9,48	9,48	9,49	9,49	9,49						
3	.25	2,46	2,46	2,46	2,47	2,47	2,47	2,47	2,47	2,47	2,47	2,47	2,47						
	.10	5,20	5,18	5,18	5,17	5,16	5,15	5,15	5,14	5,14	5,14	5,14	5,13						
4	.25	2,08	2,08	2,08	2,08	2,08	2,08	2,08	2,08	2,08	2,08	2,08	2,08						
	.10	3,87	3,84	3,83	3,82	3,80	3,80	3,79	3,78	3,78	3,77	3,76	3,76						
5	.25	1,89	1,88	1,88	1,88	1,88	1,88	1,87	1,87	1,87	1,87	1,87	1,87						
	.10	3,24	3,21	3,19	3,17	3,16	3,15	3,14	3,13	3,12	3,11	3,10	3,10						
6	.25	1,76	1,76	1,75	1,75	1,75	1,74	1,74	1,74	1,74	1,74	1,74	1,74						
	.10	2,87	2,84	2,82	2,80	2,78	2,77	2,76	2,75	2,74	2,73	2,73	2,72						
7	.25	1,68	1,67	1,67	1,66	1,66	1,65	1,65	1,65	1,65	1,65	1,65	1,65						
	.10	2,63	2,59	2,58	2,56	2,54	2,52	2,51	2,50	2,49	2,48	2,48	2,47						
8	.25	1,62	1,61	1,60	1,60	1,59	1,59	1,59	1,58	1,58	1,58	1,58	1,58						
	.10	2,46	2,42	2,40	2,38	2,36	2,35	2,34	2,32	2,31	2,30	2,29	2,29						
9	.25	1,57	1,56	1,56	1,55	1,55	1,54	1,54	1,53	1,53	1,53	1,53	1,53						
	.10	2,34	2,30	2,28	2,25	2,23	2,22	2,21	2,19	2,18	2,17	2,17	2,16						
10	.25	1,55	1,55	1,55	1,55	1,54	1,54	1,54	1,53	1,53	1,53	1,53	1,53						
	.10	2,34	2,30	2,28	2,25	2,23	2,22	2,21	2,19	2,18	2,17	2,17	2,16						
11	.25	1,54	1,54	1,54	1,54	1,53	1,53	1,53	1,52	1,52	1,52	1,52	1,52						
	.10	2,34	2,30	2,28	2,25	2,23	2,22	2,21	2,19	2,18	2,17	2,17	2,16						
12	.25	1,53	1,53	1,53	1,53	1,52	1,52	1,52	1,51	1,51	1,51	1,51	1,51						
	.10	2,34	2,30	2,28	2,25	2,23	2,22	2,21	2,19	2,18	2,17	2,17	2,16						

Source : Tiré de E. S. Pearson et H. O. Hartley, éd., *Biometrika Tables for Statisticians*, vol. 1, 3^e éd., table 18, Cambridge University Press, New York, 1966. Reproduit avec l'autorisation des auteurs et des administrateurs de Biometrika.

Table D.3 Valeurs critiques supérieures de la distribution F (suite)

df pour le dénominateur M_2	Pr	Degrés de liberté pour le numérateur M_1											
		1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1,49	1,60	1,60	1,59	1,59	1,58	1,57	1,56	1,56	1,55	1,55	1,54
	.10	3,29	2,92	2,73	2,61	2,52	2,46	2,41	2,38	2,35	2,32	2,30	2,28
	.05	4,96	4,10	3,71	3,48	3,33	3,22	3,14	3,07	3,02	2,98	2,94	2,91
11	.25	1,47	1,58	1,58	1,57	1,56	1,55	1,54	1,53	1,53	1,52	1,52	1,51
	.10	3,23	2,86	2,66	2,54	2,45	2,39	2,34	2,30	2,27	2,25	2,23	2,21
	.05	4,84	3,98	3,59	3,36	3,20	3,09	3,01	2,95	2,90	2,85	2,82	2,79
12	.25	1,46	1,56	1,56	1,55	1,54	1,53	1,52	1,51	1,51	1,50	1,50	1,49
	.10	3,18	2,81	2,61	2,48	2,39	2,33	2,28	2,24	2,21	2,19	2,17	2,15
	.05	4,75	3,89	3,49	3,26	3,11	3,00	2,91	2,85	2,80	2,75	2,72	2,69
13	.25	1,45	1,55	1,55	1,53	1,52	1,51	1,50	1,49	1,49	1,48	1,47	1,47
	.10	3,14	2,76	2,56	2,43	2,35	2,28	2,23	2,20	2,16	2,14	2,12	2,10
	.05	4,67	3,81	3,41	3,18	3,03	2,92	2,83	2,77	2,71	2,67	2,63	2,60
14	.25	1,44	1,53	1,53	1,52	1,51	1,50	1,49	1,48	1,47	1,46	1,46	1,45
	.10	3,10	2,73	2,52	2,39	2,31	2,24	2,19	2,15	2,12	2,10	2,08	2,05
	.05	4,60	3,74	3,34	3,11	2,96	2,85	2,76	2,70	2,65	2,60	2,57	2,53
15	.25	1,43	1,52	1,52	1,51	1,49	1,48	1,47	1,46	1,45	1,44	1,44	1,44
	.10	3,07	2,70	2,49	2,36	2,27	2,21	2,16	2,12	2,09	2,06	2,04	2,02
	.05	4,54	3,68	3,29	3,06	2,90	2,79	2,71	2,64	2,59	2,54	2,51	2,48
16	.25	1,42	1,51	1,51	1,50	1,48	1,47	1,46	1,45	1,44	1,44	1,44	1,43
	.10	3,05	2,67	2,46	2,33	2,24	2,18	2,13	2,09	2,06	2,03	2,01	1,99
	.05	4,49	3,63	3,24	3,01	2,85	2,74	2,66	2,59	2,54	2,49	2,46	2,42
17	.25	1,42	1,51	1,50	1,49	1,47	1,46	1,45	1,44	1,43	1,43	1,42	1,41
	.10	3,03	2,64	2,44	2,31	2,22	2,15	2,10	2,06	2,03	2,00	1,98	1,96
	.05	4,45	3,59	3,20	2,96	2,81	2,70	2,61	2,55	2,49	2,45	2,41	2,38
18	.25	1,41	1,50	1,49	1,48	1,46	1,45	1,44	1,43	1,42	1,42	1,41	1,40
	.10	3,01	2,62	2,42	2,29	2,20	2,13	2,08	2,04	2,00	1,98	1,96	1,93
	.05	4,41	3,55	3,16	2,93	2,77	2,66	2,58	2,51	2,46	2,41	2,37	2,34
19	.25	1,41	1,49	1,49	1,47	1,46	1,44	1,43	1,42	1,41	1,41	1,40	1,40
	.10	2,99	2,61	2,40	2,27	2,18	2,11	2,06	2,02	1,98	1,96	1,94	1,91
	.05	4,38	3,52	3,13	2,90	2,74	2,63	2,54	2,48	2,42	2,38	2,34	2,31
20	.25	1,40	1,49	1,48	1,46	1,45	1,44	1,43	1,42	1,41	1,40	1,39	1,39
	.10	2,97	2,59	2,38	2,25	2,16	2,09	2,04	2,00	1,96	1,94	1,92	1,89
	.05	4,35	3,49	3,10	2,87	2,71	2,60	2,51	2,45	2,39	2,35	2,31	2,28

Table D.3 Valeurs critiques supérieures de la distribution F (suite)

df pour le dénominateur M_2	Pr	Degrés de liberté pour le numérateur M_1											
		15	20	24	30	40	50	60	100	120	200	500	∞
10	.25	1,53	1,52	1,52	1,51	1,51	1,50	1,50	1,50	1,50	1,49	1,49	1,48
	.10	2,24	2,20	2,18	2,16	2,13	2,12	2,11	2,09	2,08	2,07	2,06	2,06
	.05	2,85	2,77	2,74	2,70	2,66	2,64	2,62	2,59	2,58	2,56	2,55	2,54
11	.25	1,50	1,49	1,49	1,48	1,47	1,47	1,47	1,46	1,46	1,45	1,45	1,45
	.10	2,17	2,12	2,10	2,08	2,05	2,04	2,03	2,00	2,00	1,99	1,98	1,97
	.05	2,72	2,65	2,61	2,57	2,53	2,51	2,49	2,46	2,45	2,43	2,42	2,40
12	.25	1,48	1,47	1,46	1,45	1,45	1,44	1,44	1,43	1,43	1,43	1,42	1,42
	.10	2,10	2,06	2,04	2,01	1,98	1,97	1,96	1,94	1,94	1,93	1,92	1,91
	.05	2,62	2,54	2,51	2,47	2,43	2,40	2,38	2,35	2,34	2,32	2,31	2,30
13	.25	1,46	1,45	1,44	1,43	1,42	1,42	1,42	1,41	1,41	1,40	1,40	1,40
	.10	2,05	2,01	1,98	1,96	1,93	1,92	1,90	1,88	1,88	1,86	1,85	1,85
	.05	2,53	2,46	2,42	2,38	2,34	2,31	2,30	2,26	2,25	2,23	2,22	2,21
14	.25	1,44	1,43	1,42	1,41	1,41	1,40	1,40	1,40	1,39	1,39	1,38	1,38
	.10	2,01	1,96	1,94	1,91	1,89	1,87	1,86	1,83	1,83	1,82	1,80	1,80
	.05	2,46	2,39	2,35	2,31	2,27	2,24	2,22	2,19	2,18	2,16	2,14	2,13
15	.25	1,43	1,41	1,41	1,40	1,39	1,39	1,38	1,38	1,38	1,37	1,36	1,36
	.10	1,97	1,92	1,90	1,87	1,85	1,83	1,82	1,79	1,79	1,77	1,76	1,76
	.05	2,40	2,33	2,29	2,25	2,20	2,18	2,16	2,12	2,11	2,10	2,08	2,07
16	.25	1,41	1,40	1,39	1,38	1,37	1,37	1,36	1,36	1,36	1,35	1,34	1,34
	.10	1,94	1,89	1,87	1,84	1,81	1,79	1,78	1,76	1,75	1,74	1,73	1,72
	.05	2,35	2,28	2,24	2,19	2,15	2,12	2,11	2,07	2,06	2,04	2,02	2,01
17	.25	1,40	1,39	1,38	1,37	1,36	1,35	1,35	1,34	1,34	1,34	1,33	1,33
	.10	1,91	1,86	1,84	1,81	1,78	1,76	1,75	1,73	1,72	1,71	1,69	1,69
	.05	2,31	2,23	2,19	2,15	2,10	2,08	2,06	2,02	2,01	1,99	1,97	1,96
18	.25	1,39	1,38	1,37	1,36	1,35	1,34	1,34	1,33	1,33	1,32	1,32	1,32
	.10	1,89	1,84	1,81	1,78	1,75	1,74	1,72	1,70	1,69	1,68	1,67	1,66
	.05	2,27	2,19	2,15	2,11	2,06	2,04	2,02	1,98	1,97	1,95	1,93	1,92
19	.25	1,38	1,37	1,36	1,35	1,34	1,33	1,33	1,32	1,32	1,31	1,31	1,30
	.10	1,86	1,81	1,79	1,76	1,73	1,71	1,70	1,67	1,67	1,65	1,63	1,63
	.05	2,23	2,16	2,11	2,07	2,03	2,00	1,98	1,94	1,93	1,91	1,88	1,88
20	.25	1,37	1,36	1,35	1,34	1,33	1,33	1,32	1,31	1,31	1,30	1,30	1,29
	.10	1,84	1,79	1,77	1,74	1,71	1,69	1,68	1,65	1,64	1,62	1,61	1,61
	.05	2,20	2,12	2,08	2,04	1,99	1,97	1,95	1,91	1,90	1,88	1,86	1,84

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Table D.3 Valeurs critiques supérieures de la distribution F (suite)

df pour le dénominateur M_2	Pr	Degrés de liberté pour le numérateur M_1											
		1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.26	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.26	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.26	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75

Table D.3 Valeurs critiques supérieures de la distribution F (suite)

df pour le dénominateur M_2	Pr	Degrés de liberté pour le numérateur M_1																	
		15	20	24	30	40	50	60	100	120	200	500	∞						
22	.25	1.36	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29	1.28						
	.10	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57						
	.05	2.15	2.07	2.03	1.98	1.94	1.91	1.89	1.85	1.84	1.82	1.80	1.78						
24	.25	1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.29	1.28	1.28	1.27	1.26						
	.10	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.58	1.56	1.54	1.53						
	.05	2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.80	1.77	1.75	1.73						
26	.25	1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.28	1.26	1.26	1.25	1.25						
	.10	1.76	1.71	1.68	1.65	1.61	1.59	1.57	1.56	1.55	1.53	1.51	1.50						
	.05	2.07	1.99	1.95	1.90	1.85	1.82	1.80	1.76	1.76	1.73	1.71	1.69						
28	.25	1.33	1.31	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.25	1.24	1.24						
	.10	1.74	1.69	1.66	1.63	1.59	1.57	1.56	1.53	1.52	1.50	1.49	1.48						
	.05	2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.73	1.71	1.69	1.67	1.65						
30	.25	1.32	1.30	1.29	1.28	1.27	1.26	1.25	1.25	1.24	1.24	1.23	1.23						
	.10	1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46						
	.05	2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62						
40	.25	1.30	1.28	1.26	1.25	1.24	1.23	1.23	1.22	1.21	1.21	1.20	1.19						
	.10	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38						
	.05	1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.60	1.58	1.55	1.53	1.51						
60	.25	1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15						
	.10	1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29						
	.05	1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39						
120	.25	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10						
	.10	1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19						
	.05	1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25						
200	.25	1.23	1.21	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.06						
	.10	1.52	1.46	1.42	1.38	1.34	1.31	1.28	1.24	1.22	1.20	1.17	1.14						
	.05	1.72	1.62	1.57	1.52	1.46	1.41	1.39	1.32	1.32	1.26	1.22	1.19						
∞	.25	1.22	1.19	1.18	1.16	1.14	1.13	1.11	1.10	1.09	1.08	1.06	1.06						
	.10	1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.19	1.17	1.14	1.11	1.08						
	.05	1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.24	1.17	1.11	1.00						
∞	.25	1.22	1.19	1.18	1.16	1.14	1.13	1.11	1.10	1.09	1.08	1.06	1.06						
	.10	1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.19	1.17	1.14	1.11	1.08						
	.05	1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.24	1.17	1.11	1.00						