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Epreuve : Topics in Macro 1 – Code : L3-S5-2

Instructions:

- *This is a **1h exam**. The exam consists of two parts (one on each sheet of paper).*
- *There are a total of **100 points** on the test. Please note that the number of points awarded to each question is given purely as an indication.*
- *This is a **closed-book exam**: no book, no lecture notes allowed. Calculators are allowed.*
- *You must **write legibly**, and **in a concise and precise fashion**.*

Part I - A small model of the Malthusian regime (50 points)

We consider the following, discrete-time model of joint determination of population P_t and income per capita y_t of an economy:

$$b_t = \alpha\sqrt{y_t - y_b} \tag{1}$$

$$m_t = \sqrt{y_m - y_t} \tag{2}$$

$$Y_t = \sqrt{P_t} \tag{3}$$

Equations (1) and (2) respectively define the birth rate b_t and mortality rate in this economy, for any time t. The values y_b and y_m are positive and such that $y_m = \alpha^4 y_b$; we also assume $\alpha > 1$. Income per capita is such that: $y_b \leq y_t \leq y_m$ at all time. Y_t is the gross domestic product of this economy.

1 – **(10 points)** Represent on two superposed graphes, joined by the same variable y on the x-axis:

1. the sustainable population P_t as a function of income per capita y
2. the birth and mortality rates as a function of y

2 – **(10 points)** Compute the steady-state level of income per capita $y*$, and population P^*, in this economy. Place them on the graph you did in the last question.

3 – **(15 points)** What is the law of evolution of population P_{t+1} as a function of P_t? For any starting $P_0 < P^*$, explain with your own terms how population evolves to the steady state.

4 – **(7 points)** In this model, what does an increase in α represent (for a fixed value of y_b)? What impact does it have on the steady state of the model? Comment.

5 – **(8 points)** In this model, what does an increase in y_b represent? What impact does that have on the steady state of the model? Comment.

PART II - RISKY GROWTH (50 points)

Consider a growing economy. Aggregate output in period t (Y_t) is produced by combining labor (L_t) and technology (A_t) according to

$$Y_t = A_t^{\alpha} L_t^{1-\alpha}$$

with $\alpha \in (0,1)$. Each period, the aggregate saving rate is $s \in (0,1)$, so that a fraction s of total output is saved. The output saved is invested into improving the quality of the technology (each unit of output saved in period t increases the technology in the following period, A_{t+1}). The remaining output is consumed, $C_t = (1-s)\,Y_t$.

A representative agent has the following inter-temporal preferences:

$$U_t = \mathbb{E}\left[\sum_{\tau=t}^{+\infty} \beta^{\tau-t} C_t^{\gamma}\right] = \mathbb{E}\left[\sum_{\tau=t}^{+\infty} \beta^{\tau-t} \left((1-s)\,Y_t\right)^{\gamma}\right]$$

with $\beta \in (0,1)$ and $\gamma > 0$.

There are two choices for the technological investment: a safe and a risky one.

× With the safe technological investment, each unit of output saved increases the quality of the technology for certain:

$$A_{t+1} = (1-\delta)\,A_t + sY_t$$

where $\delta \in (0,1)$ denotes the obsolescence rate of the technology (a fraction δ of the old technology becomes obsolete each period).

× With the risky technological investment, the increase in the quality of the technology from investment is random:

$$A_{t+1} = \begin{cases} (1-\delta)\,A_t & \text{with probability } 50\% \\ (1-\delta)\,A_t + 2\,(sY_t) & \text{with probability } 50\% \end{cases}$$

Half the time, the risky technological investment is useless; the other half of the time, it is twice as good as the safe investment.

Answer the following questions. If you are not perfectly certain of what the correct answer is, do not panic! Even if you just give some parts of an answer, you show that you have understood the question, and the intuition behind the economics of the question, you may get full credit. But do not make up a nonsensical answer. Just repeating what you have learned from the classnotes will not increase your grade; it may even decrease your grade if it is irrelevant.

1 – **(5 points)** In the expression for the utility function above (U_t), explain what the following terms mean: the summation sign $\left(\sum_{\tau=t}^{+\infty}\right)$, the parameter β, the powers of β (the $(\tau - t)$ exponents), the expectation sign ($\mathbb{E}[\cdot]$).

2 – **(10 points)** If $\gamma > 1$, is the representative agent risk averse (she does not like risk), risk seeking (she likes risk), or risk neutral (she's indifferent to risk)? Explain why.

3 – **(15 points)** If the agent is risk seeking (she likes risk), does she prefer the safe, the risky technology, or is the answer indeterminate? Explain why.

4 – **(20 points)** If the risky technology is used, does aggregate output (Y_t) grow faster than with the safe technology, slower than with the safe technology, or is the answer indeterminate? Explain why.