Toulouse School of Economics, 2015-2016
L3 – Topics in Macroeconomics 1 – Prof. Chaney & Batté

Exam

Instructions:

- *This is a **2h30 exam**.*
- *There are a total of **100 points** on the test. Please note that the number of points awarded to each question is given purely as an indication.*
- *This is a **closed-book exam:** no book, no lecture notes, no calculator is allowed.*
- *You must **write legibly**, and **in a concise and precise fashion**.*

Part I - A model with population growth & learning-by-doing (40 points)

Consider the following growing economy. Each period, there are L_t workers. The number of workers grows exponentially according to

$$L_{t+1} = (1+n) L_t, n > 0.$$

Each period, workers produce some output for final consumption, Y_t, using the production function

$$Y_t = A_t L_t^{\alpha}, 0 < \alpha < 1.$$

There is learning by doing, in the sense that the technology of production, A_t, depends on the stock of workers in the previous period according to

$$A_t = L_{t-1}^{\beta}, 0 < \beta < 1.$$

1 – Derive an expression for the growth rate of aggregate output, $\frac{Y_{t+1}}{Y_t}$ as a function of the population growth rate, n, and the parameters α and β. **(4 points)**

2 – Is the growth rate of this economy positive ($Y_{t+1} > Y_t$), negative ($Y_{t+1} < Y_t$), or constant ($Y_{t+1} = Y_t$)? Explain in simple words the impact of population growth (n) on the growth rate of the economy. **(4 points)**

3 – Derive an expression for the growth rate of output per capita, $\frac{Y_{t+1}/L_{t+1}}{Y_t/L_t}$, as a function of the parameters (n, α, β). **(4 points)**

4 – Under which conditions is the growth rate of output per capita positive? Explain in simple words the impact of the parameters α and β on the growth of output per capita. **(4 points)**

Now we relax the assumption of exogenous population growth rate. We assume instead that each generation, a representative agent has the following preference over consumption of the final good (C_t) and her labor supply (L_t)

$$u_t (C_t, L_t) = C_t + (24 - L_t),$$

where consumption cannot exceed output: $C_t \leq Y_t$. Note that L_t is no longer the size of the population. It is instead the number of hours the agent living in period t spends working. She decides how many hours she wants to work (L_t), taking into account that working more allows her to consume more. However, she takes as given how much the agent in the previous generation has worked $\left(A_t = L_{t-1}^{\beta}\right)$.

5 – Discuss the role of consumption and labor supply in the agent's preference. **(4 points)**

6 – Solve for the optimal choice of consumption (C_t) and labor supply (L_t) of the agent in time t. **(4 points)**

7 – If the previous generation agent worked more ($L_{t-1} \nearrow$), does the agent in period t work more, or less? Give the intuition for your result, using simple words. **(4 points)**

8 – Would the agent in period t be willing to pay the agent in period $t-1$ to work more? No equation to answer this question, just reasoning. **(4 points)**

Assume now there is a single agent who lives 2 periods (no more overlapping generations). Her intertemporal utility function is given by,

$$U = u_1(C_1, L_1) + \beta u_2(C_2, L_2) = C_1 + (24 - L_1) + \delta [C_2 + (24 - L_2)].$$

The agent maximizes her intertemporal utility subject to two resource constraints,

$$\begin{aligned} C_1 &\leq L_1^{\alpha} \\ C_2 &\leq A_2 L_2^{\alpha} \\ with\, A_2 &= L_1^{\beta} \end{aligned}$$

9 – Write down the Euler equation for labor. This equation relates the choice of labor in period 1 (L_1) to the choice of labor in period 2 (L_2). [Hint: you should use the first order condition on labor in the second period (L_2) to find this relationship]. **(4 points)**

10 – Is labor in period 2 (L_2) increasing, decreasing, or independent of labor in period 1 (L_1)? Give a few sentences to provide some intuition for this result. **(4 points)**

Part II - About land, labor supply, and fertility **(35 points)**

The fictitious land of Agraria is populated by L farmers. They live under the rule of nobles, who own the whole available land $\overline{T}$. In order to produce an amount of final good y, each farmer has to rent τ acres of land from the nobles, at a unit price of p each time period, and use their own labor h, which yields output according to the following production function:

$$y = 2h^{1/2}\tau^{1/2}$$

The unit price of the final good is normalised to 1.

Each farmer likes to have n children, but these children need to be fed and housed, which forces the farmer to buy one acre of land per child. He also cares for his consumption of the final good c, and wants to avoid working too many hours h if possible. His utility is as follows:

$$u(c, n, h) = c + \frac{3\gamma}{2} n^{2/3} - \frac{h^2}{2}$$

with $0 < \gamma < 1$ a constant.

1 – Write the budget constraint of each farmer. **(3 points)**

2 – Prove that the preferred number of children n of the farmer can be expressed as: **(2 points)**

$$n = \left(\frac{\gamma}{p}\right)^3$$

3 – Prove that the efficient land-to-labor ratio reads as: **(2 points)**

$$\frac{\tau}{h} = \frac{1}{p^2}$$

4 – Prove that each individual farmer's optimal labor supply, demand of land for production purposes, and production are respectively given by: **(5 points)**

$$h = \frac{1}{p};\ \tau = \frac{1}{p^3};\ y = \frac{2}{p^2}$$

5 – Given values for the current population L, and for land $\overline{T}$, show that the price of land p is as follows: **(3 points)**

$$p = \left[\frac{(1+\gamma^3)L}{\overline{T}}\right]^{1/3}$$

We now assume that the economy of Agraria evolves over time (denoted by t and discrete). More precisely, population evolves according to the following equation:

$$L_{t+1} = n_t L_t$$

with n_t the number of children per farmer, which is determined as before.

6 – What is the price of land at steady state? **(3 points)**

7 – Assume the following numerical values: $\overline{T} = 900$ and $\gamma = 1/2$. Show that the steady-state value of population is $L = 100$. What is the corresponding aggregate production of final good Y? **(5 points)**

8 – Starting from the steady state of the economy, what would happen if the nobles of Agraria conquered more land for their peasants (*i.e.* if T expanded)? Distinguish the effects in the short term and in the long term. Compare this situation to the model of the Malthusian regime seen in class. **(6 points)**

9 – How would the steady state be changed if γ decreased? Explain how this relates to Malthus' preventive check hypothesis. **(6 points)**

Part III - Questions about the lectures (**25 points**)

1 – **(5 points)** Consider a 3-period economy populated by a representative consumer. Let y_i ($i \in \{0, 1, 2\}$) denote the consumer's endowment of the consumption good in period i. Besides the y_i there are no other endowments in the economy. Also, assume consumers cannot produce (by working) the consumption good themselves. Finally, assume that there is no reason to purchase bonds in period 2.

Write the intertemporal budget constraint of the representative consumer. (In your solution, use R_0 to denote the real interest rate on one-period bonds purchased in period 0 and R_1 to denote the real interest rate on one-period bonds purchased in period 1.)

If the endowment in period 3 goes up ($y_3 \nearrow$), do you expect the interest rates R_0 and R_1 to go up, go down, or stay the same? No equations for this question, just simple reasoning.

2 – **(5 points)** Consider an economy with I different individuals and two types of consumption goods (apples and bananas). Let c_i^A be consumer i's consumption of apples, c_i^B consumer i's consumption of bananas. Also, let e_i^A and e_i^B denote the endowment of apples and bananas, respectively, of consumer i. Consumer i has preferences over apples and bananas given by $U_i\left(c_i^A, c_i^B\right)$, defined such that if $U_i\left(c_i^A, c_i^B\right) > U_i\left(\tilde{c}_i^A, \tilde{c}_i^B\right)$, then consumer i strictly prefers $\left(c_i^A, c_i^B\right)$ to $\left(\tilde{c}_i^A, \tilde{c}_i^B\right)$.

State, precisely and concisely, the conditions under which an allocation $\{c_1^A, c_2^A ... c_I^A, c_1^B, c_2^B, ... c_I^B\}$ is Pareto Optimal.

3 – According to the model by Cagan (1956), what are the two theoretical causes of hyperinflation? Explain which kind(s) of economic policy can effectively fight against each of these two causes. **(5 points)**

4 – This question is about the evolution of population in the very long run, and asks you to compare the historical data to the predictions made by the model seen in class (chapter 5). You can either answer each subquestion separately, or all of them at once in the form of a short essay. **(10 points)**

(a) Briefly recall the evolution over time of fertility and mortality during the first two stages of economic history.

(b) How is population growth represented in the model of unified growth theory seen in class (Strulik & Weisdorf, 2008)?

(c) Does the model accurately reproduce the historical experience of population growth, and why?