UNIVERSITÉ TOULOUSE 1 CAPITOLE

TSE
Toulouse School of Economics
École d'économie de Toulouse

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SESSION

Master 2: Environmental and natural resources economics

Natural Resource Economics
(3h00)

Thursday April 18th 2013 - 14h00 - 17h00

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Answer the two following topics:

Topic I

Discuss the validity of the Environmental Kuznet's Curve conjecture.

Topic II

The global economy produces a composite good in amount y_t at each time. The global population is assumed to be constant and normalized to one. The composite good is produced from two inputs: a man made capital stock K_t and a renewable resource, x_t denoting the resource exploitation rate. The technology is specified as:

$$y_t = f(K_t, x_t) = e^{\delta t} K_t^{\alpha} x_t^{\beta} \quad , 0 < \alpha < 1 \ , \ 0 < \beta < 1 \ , \ \alpha + \beta < 1$$

The economy thus benefits from an exogenous trend of technical progress at the rate $\delta > 0$. The renewable resource flow (think of solar energy) is limited. Let $\bar{x}$ be the upper limit upon the recoverable resource flow.

The composite good is split between consumption, c_t being the consumption rate, and investment in man made capital accumulation. The depreciation of capital is neglected. The initial level of the capital stock is given by

$K^0 > 0$. Consumption generates an instantaneous welfare defined as:

$$u(c_t) = \frac{1}{1-\epsilon} c_t^{1-\epsilon} \quad , \epsilon > 0 \ , \ \epsilon \neq 1$$

It is assumed that $\alpha < \epsilon$. The objective of the society is to maximize an infinite sum of discounted instantaneous welfare, $\rho > 0$ being the assumed constant social rate of discount.

Q1: Write the optimization problem of the society.

Q2: Denote by π_t the costate variable associated to the capital accumulation law and by γ_t the Lagrange multiplier associated to the constraint over the solar energy flow availability. Observe that $\lim_{c \downarrow 0} c^{-\epsilon} = +\infty$, $f(0,x) = f(K,0) = 0$ imply that $c_t > 0$, $x_t > 0$ and $K_t > 0$ in any optimum. Express the optimality conditions. What do you remark concerning the exploitation rate of the renewable resource? Make an economic comment.

Q3: Let $a_t = c_t/K_t$ and $b_t = y_t/K_t$. Express $\dot{K}_t/K_t$ as a function of a_t and b_t.

Q4: Making use of the optimality conditions, express the growth rate of consumption, $\dot{c}_t/c_t$. State a condition for $\dot{c}_t/c_t$ to be positive. Make an economic comment.

Q5: Using the expression of a_t and the optimality conditions, compute the growth rate of a_t, $\dot{a}_t/a_t$, as a function of a_t and b_t.

Q6: Study the locus $\dot{a} = 0$ in the (a, b) plane. Remember that by assumption $\alpha < \epsilon$. Show that this locus is a line of slope higher than one cutting the horizontal axis at $a = \rho/\epsilon$.

Q7: Using the definition of b_t, study the locus $\dot{b}_t = 0$. Show that in the (a, b) plane, this locus is a line of slope equal to one starting from $b(0) = \delta/(1-\alpha) > 0$. Conclude to the existence of a unique steady state $(\hat{a}, \hat{b})$.

Q8: Check that at the steady state: $\dot{c}_t/c_t = \dot{y}_t/y_t = \dot{K}_t/K_t$. Compute the expression of the common asymptotic growth rate of c_t, y_t and K_t when approaching the steady state and check that it should be positive.

Q9: Picture the dynamics of a_t and b_t in the (a, b) plane using the phase diagram technique. Check the existence of two saddle branches converging toward the steady state $(\hat{a}, \hat{b})$. Make a comment about the possible optimal dynamics of a and b taking into account that $b_0 = (K^0)^{\alpha-1}\bar{x}^{\beta}$ defines b_0 as a decreasing function of the initial capital stock K^0. Check that the consumption rate c_t increases with the availability of the solar resource, that is with $\bar{x}$.

Q10: Assume that K^0 is such that $b_0 > \hat{b}$. Show that along the optimal path, $\dot{K}_t > 0$ and $\dot{c}_t > 0$ (**Hint:** Use the results of Q3 for the dynamics of K_t and the results of Q4 for those of c_t).

Q11: Compute the dynamics of γ_t, the opportunity cost of the solar availability constraint, along the optimal path in a case where $b_0 > \hat{b}$. What do you observe? Make an economic comment.