MICROECONOMICS II A

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2 hours

Closed books

Questions (5 points)

Answer the following questions and *briefly* comment on them.

1. Consider the implementation problem with a universal domain of preferences and more than two alternatives. What social choice functions can be strongly implemented in dominant strategies? In Nash equilibrium? Is it possible to do better in Nash equilibrium than in dominant strategies?

2. Is it ever useful to consider indirect revelation mechanisms?

3. Does a risk-averse seller prefer a second-price auction or a first-price auction?

4. Consider an adverse selection setting with quasi-linear utilities, where the agent's objective is of the form $U(q;\theta)+t$, and two types, $\underline{\theta}$ and $\bar{\theta}\,(>\underline{\theta})$. Under which conditions are the relevant constraints: (i) the participation constraint of a *"bad"* type, and (ii) the incentive constraint of a *"good"* type?

5. In a moral-hazard setting, is it always optimal to condition the compensation of the agent on all available signals?

Problem: Monopolistic provision of insurance (15 points)

Framework

A monopolistic insurance provider (the principal) proposes an insurance contract to a household (the agent). The household has initially a financial wealth $w>0$, but can have an accident with probability θ, in which case s/he incurs a loss L, where $0<L<w$.

The utility function of the household over wealth levels is

$$u: \begin{array}{ccc} \mathbb{R}_+ & \rightarrow & \mathbb{R} \\ w & \rightarrow & u(w) \end{array},$$

and is assumed to satisfy the following conditions:

- it is twice continuously differentiable, strictly increasing and strictly concave: $u'(\cdot) > 0 > u''(\cdot)$;
- $u'(0) = +\infty$.

The insurance contract can specify an insurance premium, t, and the percentage of losses that will be covered in case of an accident, α. The principal's objective is thus

$$V(t, \alpha; \theta) = t - \theta \alpha L \quad (1)$$

whereas the θ-type agent's objective is

$$U(t, \alpha; \theta) = \theta u(w - t - (1 - \alpha) L) + (1 - \theta) u(w - t) \quad (2)$$

It is assumed that over-insurance is forbidden, so that we must have $\alpha \in [0, 1]$.

1. Is there any loss of generality in restricting attention to contracts described by (t, α)?

2. Express the objectives of the principal and the agent as a function of (w_N, w_L), where $w_N = w - t$ denotes the agent's wealth in the absence of an accident, and $w_L = w - t - (1 - \alpha) L$ denotes the agent's wealth in the case of accident.

3. Represent the iso-profit curves of the principal (V constant) and the iso-utility curves of the agent (U constant) in the (w_N, w_L) plane. What are their slopes? How do they vary with θ?

Complete information

We suppose in this section that θ is known by both the principal and the agent. The principal makes a take-it-or-leave-it offer, which the agent accepts of rejects. If the agent rejects the offer, s/he obtains an expected utility equal to:

$$\hat{U}(\theta) = \theta u(w - L) + (1 - \theta) u(w).$$

4. Characterize the optimal contract $\left(t^{FB}(\theta), \alpha^{FB}(\theta)\right)$, or equivalently $\left(w_N^{FB}(\theta), w_L^{FB}(\theta)\right)$, under complete information ("first-best"). Illustrate it graphically in the (w_N, w_L) plane and comment.

5. How does the optimal contract vary with θ?

Incomplete information

We now supose that the agent's type θ is only known by the agent, and not by the principal. The probability θ can take two values: $\theta \in \Theta \equiv \{\underline{\theta}, \overline{\theta}\}$, with $0 < \underline{\theta} < \overline{\theta} < 1$. The principal has prior probabilities $\Pr(\theta = \underline{\theta}) = \nu$ and $\Pr(\theta = \overline{\theta}) = 1 - \nu$.

6. Is the first-best contract $\left\{\left(w_N^{FB}(\theta), w_L^{FB}(\theta)\right)\right\}_{\theta\in\Theta}$ still implementable? Why or why not?

7. Feasibility constraints.

   (a) Write down the agent's incentive and participation constraints, using $(\underline{w}_N, \underline{w}_L) = (w_N(\underline{\theta}), w_L(\underline{\theta}))$ and $(\bar{w}_N, \bar{w}_L) = \left(w_N(\bar{\theta}), w_L(\bar{\theta})\right)$ as relevant variables, and $(\underline{U}, \bar{U}) = \left(\hat{U}(\underline{\theta}), \hat{U}(\overline{\theta})\right)$ for the agent's outside options.

   (b) Illustrate the incentive constraints in the (w_N, w_L) plane.

   (c) Does α necessarily need to be increasing in θ? What about t?

8. Characterization of the second-best contract $\{t^{SB}(\theta), \alpha^{SB}(\theta)\}_{\theta\in\Theta}$, or equivalently $\left\{\left(\underline{w}_N^{SB}, \underline{w}_L^{SB}\right), \left(\bar{w}_N^{SB}, \bar{w}_L^{SB}\right)\right\}$.

   (a) Intuitively, which constraints should bind and which should be slack?

   (b) Write the first-order conditions (if you use a Lagrangian, denote by λ and μ the multipliers associated with the relevant constraints). Are these first-order conditions necessary? Sufficient?

   (c) Is there full insurance for $\underline{\theta}$? For $\overline{\theta}$?

   (d) Are the neglected constraints indeed satisfied?

   (e) Illustrate the second-best contracts $\left\{\left(\underline{w}_N^{SB}, \underline{w}_L^{SB}\right), \left(\bar{w}_N^{SB}, \bar{w}_L^{SB}\right)\right\}$ in the (w_N, w_L) plane.

9. Limited participation

   (a) Can it be optimal to exclude one type? both types?

   (b) If so, under which condition(s)?

Countervailing incentives

Assume that the $\overline{\theta}$-type has a reservation utility level

$$\overline{U} = \overline{\theta} u(w - L) + \left(1 - \overline{\theta}\right) u(w) + \Delta,$$

where $\Delta > 0$.

10. Discuss qualitatively how the second-best will evolve with Δ. Which constraints will be relevant?

11. Suppose that the utility function is $u(x) = \log(x)$. Characterize the second-best contract for different levels of $\overline{U}$.

Continuous types (optional question)

This question is optional. It is not needed to obtain the top grade but can be used as a bonus, to compensate other questions.

Return to the case when $\Delta = 0$, but assume now that θ is continuously distributed over $[0, 1]$ with distribution $F(\cdot)$ and strictly positive density $f(\cdot) > 0$ for all $\theta \in [0, 1]$.

12. Characterize the second-best contract. (**Hint:** make the appropriate assumptions on the hazard rate if necessary).