Macro Economics 2 (M2 Eco-Math): Final Exam, April 15, 2013

Instructions:

There are two parts to the exam. In the first Part, you are given 5 short-answer questions, of which you must answer 4. This part counts for 40% of your exam. The second part consists of three longer problems to solve, of which you must answer 2. This part counts for 60% of the exam.

Part 1:

Question 1a: In a celebrated article, Lucas (2003) analyzed the welfare costs of business cycle fluctuations, and provided a numerical estimate suggesting that these are very small. What is the basis for the argument, i.e. the key assumptions that lead to his conclusion. Do you agree with his argument and conclusion?

Question 1b: Do you agree with the following statement: "A welfare-maximizing monetary policy should aim to keep nominal interest rates low and stable, and not take an active stance to respond to aggregate or price level fluctuations".

Question 1c: Which of the models you have seen in this course would you use or adapt to analyze the following:

(i) the patterns of earnings dynamics and income and wealth inequality

(ii) the sharing of business cycle risks across member states of the European Union

(iii) the welfare costs associated with a large disinflation policy, such as the one implemented by Volcker in 1980?

(iv) the increase in personal bankruptcies and mortgage defaults that we have seen in the US over the last 5-10 years

(v) the design of payroll taxes and unemployment insurance schemes

Question 1d: Suppose that a security has stochastic return $\widetilde{R}_t^j = R(z^t) + e_t^j$, where e_t^j is an idiosyncratic, security specific factor with $cov\left(e_t^j, m(z^t)\right) = 0$, and $R(z^t)$ is the expected return of the security conditional on an aggregate state z^t. (for example, think of decomposing the return on an individual stock $\widetilde{R}$ into a market return $R(z^t)$ and an idiosyncratic component e_t^j). Derive the following modified version of the Hansen-Jagannathan bound:

$$\frac{E\left(\widetilde{R}_t^j\right) - R_F}{\sigma\left(\widetilde{R}_t^j\right)} \frac{\sigma\left(\widetilde{R}_t^j\right)}{\sigma\left(R\left(z^t\right)\right)} \leq R_F \sigma\left(m\left(z^t\right)\right).$$

where $\sigma\left(\widetilde{R}_t^j\right)/\sigma\left(R\left(z^t\right)\right) = \sqrt{\sigma^2\left(R\left(z^t\right)\right) + \sigma^2\left(e_t^j\right)}/\sigma\left(R\left(z^t\right)\right) > 1$. How does this calculation modify the original bound, and for what securities will this modification be important?

Question 1e: A corporate bond of a certain rating category has on average a default rate of 0.5% (you may assume that in case of default there is a 100% loss, i.e. nothing is recovered), yet the "yield spread" (i.e. the promised return over and above the risk-free rate) is 3%.

The aggregate default rate, while averaging to 1% is also time varying, and equal to 0% w.p. 1/2 and 1% w.p. 1/2.

(i) Using the formula derived under Question 1d, compute a Sharpe Ratio bound for this corporate bond; (hint: remember that if x is a binary random variable with support on 0 and 1, then $Var(x) = p(1-p)$, where $p = \Pr(x = 1)$).

(ii) The numbers you were given are roughly representative of bond returns in the data. What do you conclude from this exercise, in comparison with the Sharpe ratio bounds we computed for equity returns?

Part 2:

Problem 1: A planning problem with heterogeneous firms

Consider the following social planner's problem (notation and any element not defined here are as in the lectures). Time is discrete and infinite. There is a representative household, with preferences defined as in the course, over consumption and labor supply sequences.

There is a measure 1 of firms. A firm j has technology

$$y^j\left(s^t\right) = A^j\left(s^t\right) f\left(k^j\left(s^{t-1}\right), n^j\left(s^t\right)\right),$$

where $s^t = \left(z^t, h^t\right)$ consists of an idiosyncratic and an aggregate part. $y^j\left(s^t\right)$ denotes the firm's output of the consumption good, while $k^j\left(s^{t-1}\right)$ is its capital input (determined in the previous period) and $n^j\left(s^t\right)$ its labor input. Capital depreciates at a rate δ. The planner's allocation determines the capital and labor allocations at each firm for all s^t, as well as the households' consumption and labor supply at each aggregate state z^t.

(i) Write down the social planner's problem for this production economy.

(ii) Take first-order conditions. What can you say about the optimal allocation of productive inputs across firms?

(iii) What can you say about the equilibrium prices that support the efficient allocation?

(iv) Under what conditions do you obtain a separation result, under which aggregate output can be represented by an aggregate production function of the form

$$Y\left(z^t\right) = A\left(z^t\right) F\left(K\left(z^{t-1}\right), N\left(z^t\right)\right)$$

and separated from the allocation of labor and capital inputs at the firm level?

(v) Suppose you are given a panel data set of the form $\left(y_t^i, k_t^i, n_t^i\right)$ of output, capital input and labor input for firm i in period t. How would you test for the implications of the planner's solution in your data?

Problem 2: Investment with fixed costs of adjustment

Time is continuous and infinite. There is a continuum of measure 1 of firms, who all discount the future at a constant (net) interest rate r. The firms have a strictly concave production technology whose flow output is $f(k) = k^\alpha$ (where $\alpha \in (0,1)$), and their capital depreciates at a rate $\delta \in (0,1)$. The firms face fixed costs $F > 0$ of investment. Specifically, the cost of installing i units of new capital is

$$C(i) = \begin{cases} 0 \text{ if } i = 0 \\ F + i \text{ if } i > 0 \end{cases}.$$

(i) State the dynamic optimization problem for an individual firm.

(ii) Show that at the optimum, the firms make fixed investments of a constant size at regular time intervals.

(iii) Consider now a stationary environment in which the firms' investments are uniformly staggered across time. Suppose that there is a sudden depreciation shock, by which all capital depreciates by a factor $\Delta > 0$ (small). What are the effects of this depreciation shock for investment on impact, and over time? What about the aggregate capital stock, and what about output?

(iv) What is the role of the returns to scale assumption for the solution to this problem?

...incomplete...

...st decade there has be...
...ces" in capital flows: some...
...owing from foreign countries, whi...
China, are experiencing very large and...
puzzling is that the lending countries are...
rates. Here we explore a simple model to ...
imbalances.

There are two periods, and two countries, U a...
measure of households, whose preferences over con...
non-storable consumption good are given by

$$\log c_0 + \beta \log c_1.$$

Each household starts at date 0 with a wealth of 1, which...
risky investment (k_0^i) that yields an idiosyncratic, stochast...
...ase financial claims. The return A^i is stochastic, iid across ...
idiosyncratic risk!), and takes on either of two values, A_j...
$\pi_{j,L}$, respectively. The subscript $j = U, C$ stands for the ...
...s to be possibly different across countries, but generally ...
...ed return than U. The two countries also differ in the finan...

...a financially developed country: Here, agents can agree o...
...urance contracts against the idiosyncratic investment risk...
...ore limited financial market: only risk-free bonds can be tr...

...e two countries are in financial autarky (i.e. they cann...
...he border).

...ial market equilibrium in country U. What are consumpt...
...e rate on bonds in country U?

...market equilibrium in country C. What are consumption
...rate on bonds in country C?

...s become financially integrated, and can trade risk-
... < 1, so that idiosyncratic risk plays no role in
...to investment. Characterize the two-country
...rrows, and who lends? What is the risk-free
...ize the two-country equilibrium w...
...nds?

...the lens of this simpl...