Jean-Pierre FLORENS - Econometrics II - April 2013

I - Let us assume that a consumer has an utility $u(Q,\theta) - Q\varepsilon$ for a quantity Q of some good. The function u is known up to some parameters θ and ε is an heterogeneity component distributed with some probability measure. The consumer is faced to a non linear pricing rule $P(Q)$ assumed to be differentiable and known by the agents and by the econometrician. Then the optimal decision for the consumer is obtained by the resolution of

$$V(Q,\theta) - p(Q) = \varepsilon$$

where $V(Q,\theta) = \frac{\partial}{\partial Q} u(Q,\theta)$ and $p(Q) = \frac{\partial}{\partial Q} P(Q)$

i) For iid sample of consumer the $\varepsilon_i (i = 1, ..., n)$ are non observable but we assume that there are $(N(0,\sigma^2)$. We observe the sample $Q_i (i = 1, ..., n)$. Write the likelihood of the sample given the parameters θ and σ^2. What are the properties of the MLE of θ and σ^2 and give the asymptotic variance of these estimators.

ii) Assume that θ is given and that the interest is focused on the σ parameter which describes the heterogeneity of the consumer. What is a natural conjugate prior on σ (you may parametrize the model by σ^2 or by $\frac{1}{\sigma^2}$) and what in the posterior distribution ? Let us denote the prior density by $\nu(\sigma)$. If now θ and σ are unknown and provided by a prior density $\mu(\theta)\nu(\sigma)$ how do you compute the posterior distribution on θ ?

iii) We keep the assumption that the ε_i are iid but we ignore their distribution. The distributional assumption is now replaced by the introduction of some instruments. Let $Z \in I\!R^p$ a vector of observed variables (possibly including the constant 1) and we assume that $E(\varepsilon|Z) = 0$.
Explain how to estimate θ by GMM under the assumption $Var(\varepsilon|Z) = \sigma^2 \quad \forall Z$. Consider the case where the optimal weighting matrix is used and explain how you estimate this matrix. Discuss also the choice of the optimal instruments.
As the optional instruments involve $E(\frac{\partial}{\partial \theta} V(Q,\theta)|Z)$ we propose a non parametric estimation of this conditional expectation.
Explain how you may construct this estimator by a kernel smoothing method and how this estimator may be used in the GMM approach.

iv) We now assume that the distribution of ε is known and we want to relax the parametric specification of u.
Let us write the model :

$$\varphi(Q) = \varepsilon \text{ where } \varphi(Q) = V(Q) - p(Q)$$

We first want to estimate $\varphi(Q)$ using the iid sample $(Q_i|i = 1, ..., n)$. Assuming φ strictly monotone and decreasing how can we estimate φ from the relation between the c.d.f. of Q and the c.d.f. of ε. (We assume that ε has a continuous distribution such that its c.d.f. is invertible).

v) We now consider a continuous variable X such that V becomes a function $V(Q, X))$ and V is treated non parametrically. Assume that we observe two prices regimes $p^1(Q)$ and $p^2(Q)$. We want to show that in that case V is non parametrically identifiable. Let $G(q|x) = Prob(Q \leq q|X = x)$. Show that $\frac{\partial G}{\partial q} / \frac{\partial G}{\partial x}$ does not depend on the distribution of ε and deduce from this equality the identification result.

II - Let X an univariate random variable with a density f and $x_1, ..., x_n$ an iid sample. We estimate f by $\hat{f}$ the kernel estimate. Show that the bootstrap distribution of $\hat{f}$ has a mean equal to $\hat{f}$.