

Master 2 ERNA - MIF - Eco Maths
Economics of Risk and Uncertainty
TSE
December 2012

Calculators provided by the University are allowed.
Please answer to Part I and Part II on different sheets.

Part I

Exercise 1

An individual has a VNM utility function $u(\cdot)$ concave and initial wealth w . Let \tilde{x} be a favourable lottery.

- 1) If the individual owns the lottery, what is the minimum price $p_s(w)$ he would sell it for?
- 2) If he does not own it, what is the maximum price $p_b(w)$ he would be willing to pay for it?
- 3) Show that buying and selling prices are equal only if $p_s(w)$ is independent of wealth.
- 4) What is the condition on the utility function that ensures that $p_s(w)$ is independent of wealth?

Exercise 2

A lottery gives 100 with probability 1/2 and -100 with probability 1/2. A risk averse agent without any initial wealth wonders whether he should run one lottery, or two lotteries simultaneously. Can you make a recommendation?

Exercise 3

We consider an insurance market that lasts for two periods. At each date, agents have initial wealth w and face a probability p of an accident which results in a loss L . Agents cannot borrow or save. At each date ($t = 1, 2$), an agent has a VNM utility $u(x_t)$ where x_t represents the net wealth of the current period. Insurance is compulsory : all agents are obliged to buy a full insurance contract at each date.

Insurance companies offer long-term full insurance contracts that consist of a premium P_1 for the first period and a premium for the second period that depends on the first period accident. In the second period, agents pay P_2^a in case of an accident in the first period or pay P_2^n in case of

no accident. P_1 , P_2^a and P_2^n are all determined in the first period. There is perfect competition in the insurance market.

The discount factor is 1 for both parties (insurers and agents).

We assume that no party knows the true value of p at the beginning of the relationship. It is initially independently distributed in the population with mean p_0 . Based on the first period information on accidents, all the parties revise the beliefs on p at the end of the first period. Denote p_a the posterior mean of p given that the agent had an accident and p_n the posterior mean of p given that the agent had no accident. We have

$$p_n < p_0 < p_a$$

$$p_0 = p_0 p_a + (1 - p_0) p_n.$$

- 1) Assume that both insurers and agents are committed to the contract. Determine the competitive equilibrium contract $(P_1^*, P_2^{a*}, P_2^{n*})$. (Do not forget to look at the insurer's profit.)
- 2) Assume now that only insurers are committed to the contract : at the second period, the agent can break the contract with the insurer without any penalty and buy a one period insurance contract from another insurer. The history of accident of an agent is publicly observable.
 - (a) Show that with the contract derived in question 1), an agent who has no accident in the first period will break his contract.
 - (b) What is the maximal premium P_2^n that an insurer can propose in the long term first period contract ?
 - (c) In equilibrium, all the agents buy a long term contract in the first period and stay with their insurer. Derive this contract and show that it involves a bonus but no malus.
- 3) Assume now that in the context of question 2, the history of accident is only known by the agent and his insurer but not by competing insurers. What is the equilibrium long-term contract?

Part II

Exercise 1:

Consider a certain economy with a growth rate g . Demonstrate that the Ramsey rule implies that the discount rate can be interpreted as the combination of two effects : an impatience effect and a wealth effect. Specify the assumptions that you are considering.

Exercise 2:

The representative agent has a rate of impatience $\delta = 0$ and a constant relative risk aversion $\gamma = 2$. He believes that the growth of the economy follows a random walk. Describe the term structure of discount rates in the following two cases:

1. The annual growth rate is either 0% or 4% with equal probabilities.
2. The annual growth rate is either 0% with probability 1/3, and 3% with probability 2/3.

Suppose now that the representative agent has diminishing expectations on the growth of the economy i.e. that the growth of the economy $g_t = (1 - \Phi)g_{t-1} + \Phi\mu$ where μ is the average growth. Thus, the term structure of discount rates is given by the following condition:

$$r_t = \delta + \gamma \frac{\ln c_t - \ln c_0}{t}$$

moreover the agent consumption at date $t+1$, $c_{t+1} = c_t e^{g_t}$.

Describe the term structure of the discount rates in the following two cases :

3. $\mu = 2\%$, $\Phi = 0.3$ and $g_0 = 4\%$
4. $\mu = 2\%$, $\Phi = 0.3$ and $g_0 = 2\%$